About This Book

Welcome to *Prealgebra*, an OpenStax resource. This textbook has been created with several goals in mind: accessibility, customization, and student engagement—all while encouraging students toward high levels of academic scholarship. Instructors and students alike will find that this textbook offers a strong foundation in pre-algebra in an accessible format.

About OpenStax

OpenStax is a non-profit organization committed to improving student access to quality learning materials. Our free textbooks go through a rigorous editorial publishing process. Our texts are developed and peer-reviewed by educators to ensure they are readable, accurate, and meet the scope and sequence requirements of today's college courses. Unlike traditional textbooks, OpenStax resources live online and are owned by the community of educators using them. Through our partnerships with companies and foundations committed to reducing costs for students, OpenStax is working to improve access to higher education for all. OpenStax is an initiative of Rice University and is made possible through the generous support of several philanthropic foundations. OpenStax textbooks are used at many colleges and universities around the world. Please go to https://openstaxcollege.org/pages/adoptions to see our rapidly expanding number of adoptions.

About OpenStax's Resources

OpenStax resources provide quality academic instruction. Three key features set our materials apart from others: they can be customized by instructors for each class, they are a "living" resource that grows online through contributions from educators, and they are available free or for minimal cost.

Curation

To broaden access and encourage community curation, *Prealgebra* is “open source” licensed under a Creative Commons Attribution (CC-BY) license. The mathematics community is invited to submit feedback to enhance and strengthen the material and keep it current and relevant for today’s students.

Cost

Our textbooks are available for free online, and in low-cost print and e-book editions.

About Prealgebra

The core of *Prealgebra* is a textbook for a one-semester course that serves as a bridge between arithmetic and algebra. The basic philosophy of this book is to strengthen students’ arithmetic skills and introduce the fundamental concepts and vocabulary of algebra in a nurturing, non-threatening environment while addressing the needs of students with diverse backgrounds and learning styles.

Coverage and Scope

Openstax Prealgebra follows a nontraditional approach in its presentation of content. The beginning, in particular, is presented as a sequence of small steps so that students gain confidence in their ability to succeed in the course. The order of topics was carefully planned to emphasize the logical progression throughout the course and to facilitate a thorough understanding of each concept. As new ideas are presented, they are explicitly related to previous topics.

Chapter 1: Whole Numbers

Each of the four basic operations with whole numbers–addition, subtraction, multiplication, and division–is modeled and explained. As each operation is covered, discussions of algebraic notation and operation signs, translation of algebraic expressions into word phrases, and the use the operation in applications are included.

Chapter 2: The Language of Algebra

Mathematical vocabulary as it applies to the whole numbers is presented. The use of variables, which distinguishes algebra from arithmetic, is introduced early in the chapter, and the development of and practice with arithmetic concepts use variables as well as numeric expressions. In addition, the difference between expressions and equations is discussed, word problems are introduced, and the process for solving one-step equations is modeled.

Chapter 3: Integers

While introducing the basic operations with negative numbers, students continue to practice simplifying, evaluating, and translating algebraic expressions. The Division Property of Equality is introduced and used to solve one-step equations.

Chapter 4: Fractions

Fraction circles and bars are used to help make fractions real and to develop operations on them. Students continue simplifying and evaluating algebraic expressions with fractions, and learn to use the Multiplication Property of Equality to solve equations involving fractions.

Chapter 5: Decimals

Basic operations with decimals are presented, as well as methods for converting fractions to decimals and vice versa. Averages and probability, unit rates and unit prices, and square roots are included to provide opportunities to use and round decimals.

Chapter 6: Percents

Conversions among percents, fractions, and decimals are explored. Applications of percent include calculating sales tax, commission, and

simple interest. Proportions and solving percent equations as proportions are addressed as well.

Chapter 7: The Properties of Real Numbers

The properties of real numbers are introduced and applied as a culmination of the work done thus far, and to prepare students for the upcoming chapters on equations, polynomials, and graphing.

Chapter 8: Solving Linear Equations

A gradual build-up to solving multi-step equations is presented. Problems involve solving equations with constants on both sides, variables on both sides, variables and constants on both sides, and fraction and decimal coefficients.

Chapter 9: Math Models and Geometry

The chapter begins with opportunities to solve “traditional” number, coin, and mixture problems. Geometry sections cover the properties of triangles, rectangles, trapezoids, circles, irregular figures, the Pythagorean Theorem, and volumes and surface areas of solids. Distance-rate-time problems, and formulas are included as well.

Chapter 10: Polynomials

Adding and subtracting polynomials is presented as an extension of prior work on combining like terms. Integer exponents are defined and then applied to scientific notation. The chapter concludes with a brief introduction to factoring polynomials.

Chapter 11: Graphs

This chapter is placed last so that all of the algebra with one variable is completed before working with linear equations in two variables. Examples progress from plotting points to graphing lines by making a table of solutions to an equation. Properties of vertical and horizontal lines and intercepts are included. Graphing linear equations at the end of the course gives students a good opportunity to review evaluating expressions and solving equations.

All chapters are broken down into multiple sections, the titles of which can be viewed in the Table of Contents.

Accuracy of the Content

We have taken great pains to ensure the validity and accuracy of this text.

The Solutions Manual, which was written and developed after the Student Edition, has also been rigorously checked for accuracy.

In spite of the efforts described above, we acknowledge the possibility that —as with any textbook—some errata have slipped past the guards. We encourage users to report errors via our Errata (https://openstaxcollege.org/textbooks/prealgebra/errata) page.

About Our Team

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Lynn Marecek and MaryAnne Anthony-Smith have been teaching mathematics at Santa Ana College for many years and have worked together on several projects aimed at improving student learning in developmental math courses. They are the authors of *Strategies for Success: Study Skills for the College Math Student*, published by Pearson HigherEd.

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Lynn Marecek has focused her career on meeting the needs of developmental math students during her tenure as a professor at Santa Ana College and prior to that as a high school teacher. No matter what she is involved in, her passion is her students-especially those who have struggled with math. At Santa Ana College she has been awarded the Distinguished Faculty Award, Innovation Award, and the Curriculum Development Award four times. She is a Coordinator of Freshman Experience Program, the Department Facilitator for Redesign, and a member of the Student Success and Equity Committee and the Basic Skills Initiative Task Force. She is a member of the AMATYC Developmental Math Committee. Lynn holds a bachelor's degree from Valparaiso University and master's degrees from Purdue University and National University. She has also completed work in

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MaryAnne Anthony-Smith was a mathematics professor at Santa Ana College for 39 years, until her retirement in June 2015. She was honored to have been hired fresh out of graduate school as the first woman math professor at the college and has since been honored with the college's Distinguished Faculty, Professional Development, Curriculum Development, and Professional Achievement awards. MaryAnne has served as department chair, acting dean, chair of the professional development committee, institutional researcher, and faculty coordinator on several state and federally-funded grants. She is the community college coordinator of California's Mathematics Diagnostic Testing Project, a member of AMATYC's Placement and Assessment Committee and is on the math workgroup writing the California Community Colleges' Common Assessment. She is the author of the chapter *Making Children Count: Fostering Mathematical Literacy* in the 1999 NCTM publication *Developing Mathematically Promising Students*. She earned her bachelor's degree from the University of California San Diego and master's degrees from San Diego State and Pepperdine Universities.

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Introduction to Whole Numbers
By the end of this section, you will be able to:

- Identify counting numbers and whole numbers
- Model whole numbers
- Identify the place value of a digit
- Use place value to name whole numbers
- Use place value to write whole numbers
- Round whole numbers

Identify Counting Numbers and Whole Numbers

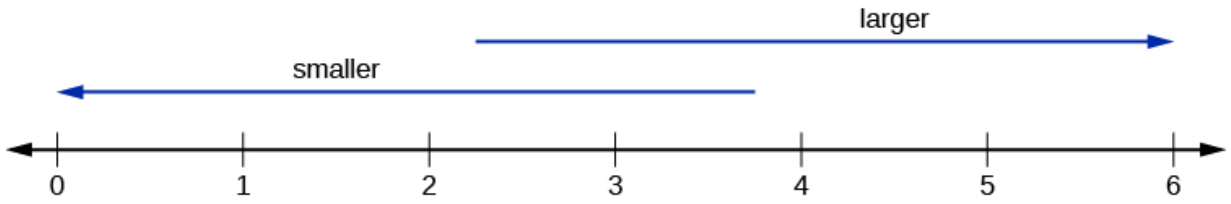
Learning algebra is similar to learning a language. You start with a basic vocabulary and then add to it as you go along. You need to practice often until the vocabulary becomes easy to you. The more you use the vocabulary, the more familiar it becomes.

Algebra uses numbers and symbols to represent words and ideas. Let's look at the numbers first. The most basic numbers used in algebra are those we use to count objects: $1, 2, 3, 4, 5$, and so on. These are called the **counting numbers**. The notation "…" is called an **ellipsis**, which is another way to show "and so on", or that the pattern continues endlessly. Counting numbers are also called **natural numbers**.

Note:
Counting Numbers
The counting numbers start with 1 and continue.
Equation:

$$\{1, 2, 3, 4, 5 \ldots\}$$

Counting numbers and whole numbers can be visualized on a **number line** as shown in [link].



The numbers on the number line increase from left to right, and decrease from right to left.

The point labeled 0 is called the **origin**. The points are equally spaced to the right of 0 and labeled with the counting numbers. When a number is paired with a point, it is called the **coordinate** of the point.

The discovery of the number zero was a big step in the history of mathematics. Including zero with the counting numbers gives a new set of numbers called the **whole numbers**.

Note:
Whole Numbers
The whole numbers are the counting numbers and zero.
Equation:

$$\{0, 1, 2, 3, 4, 5\ldots\}$$

Example:
Exercise:

Problem:

Which of the following are (a) counting numbers? (b) whole numbers?

$0, \frac{1}{4}, 3, 5.2, 15, 105$

Solution:

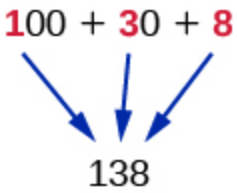
(a) The counting numbers start at 1, so 0 is not a counting number. The numbers $3, 15,$ and 105 are all counting numbers

(b) Whole numbers are counting numbers and 0. The numbers $0, 3, 15,$ and 105 are whole numbers.

The numbers $\frac{1}{4}$ and 5.2 are neither counting numbers nor whole numbers. We will discuss these numbers later.

Model Whole Numbers

Our number system is called a **place value system** because the value of a digit depends on its position, or place, in a number. The number 537 has a different value than the number 735. Even though they use the same digits, their value is different because of the different placement of the and the 7 and the 5.



Place value	Digit	Total value
hundreds	1	100
tens	3	30
ones	8	8
		138

Identify the Place Value of a Digit

A **place value chart** is a useful way to summarize place values. The place values are separated into groups of three, called periods. The periods are *ones*, *thousands*, *millions*, *billions*, *trillions*, and so on. In a written number, commas separate the periods.

[link] shows how the number 5,278,194 is written in a place value chart.

Place Value														
Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
								5	2	7	8	1	9	4

- The digit 5 is in the millions place. Its value is $5{,}000{,}000$.
- The digit 2 is in the hundred thousands place. Its value is $200{,}000$.
- The digit 7 is in the ten thousands place. Its value is $70{,}000$.
- The digit 8 is in the thousands place. Its value is $8{,}000$.
- The digit 1 is in the hundreds place. Its value is 100.
- The digit 9 is in the tens place. Its value is 90.
- The digit 4 is in the ones place. Its value is 4.

Note:
Exercise:

Problem:

For each number, find the place value of digits listed: $27{,}493{,}615$

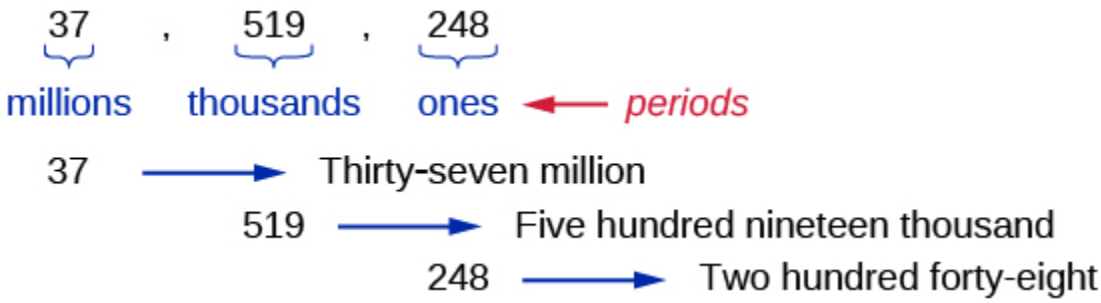
$(\mathrm{a})\ 2\ ;(\mathrm{b})\ 1\ ;(\mathrm{c})\ 4\ ;(\mathrm{d})\ 7\ ;(\mathrm{e})\ 5$

Solution:

(a) ten millions (b) tens (c) hundred thousands (d) millions (e) ones

Use Place Value to Name Whole Numbers

When you write a check, you write out the number in words as well as in digits. To write a number in words, write the number in each period followed by the name of the period without the ‘s’ at the end. Start with the digit at the left, which has the largest place value. The commas separate the periods, so wherever there is a comma in the number, write a comma between the words. The ones period, which has the smallest place value, is not named.



The number 37,519,248 is written thirty-seven million, five hundred nineteen thousand, two hundred forty-eight.

Notice that the word *and* is not used when naming a whole number.

Note:
Name a whole number in words.
Step 1. Starting at the digit on the left, name the number in each period, followed by the period name. Do not include the period name for the ones.
Step 2. Use commas in the number to separate the periods.

Example:
Exercise:

Problem: Name the number 8,165,432,098,710 in words.

Solution:

Eight trillion, one hundred sixty-five billion, four hundred thirty-two million, ninety-eight thousand, seven hundred ten.

Note:
Exercise:

Problem: Name each number in words: $9{,}258{,}137{,}904{,}061$

Solution:

Nine trillion two hundred fifty-eight billion one hundred thirty-seven million nine hundred four thousand sixty-one

Example:
Exercise:

Problem:

A student conducted research and found that the number of mobile phone users in the United States during one month in 2014 was $327{,}577{,}529$. Name that number in words.

Solution:

The number of mobile phone users in the Unites States during the month of April was three hundred twenty-seven million, five hundred seventy-seven thousand, five hundred twenty-nine.

Note:
Exercise:

Problem: One year is $31{,}536{,}000$ seconds. Name that number.

Solution:

Thirty one million, five hundred thirty-six thousand

Use Place Value to Write Whole Numbers

We will now reverse the process and write a number given in words as digits.

Note:
Use place value to write a whole number.
Step 1. Identify the words that indicate periods. (Remember the ones period is never named.)
Step 2. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.
Step 3. Name the number in each period and place the digits in the correct place value position.

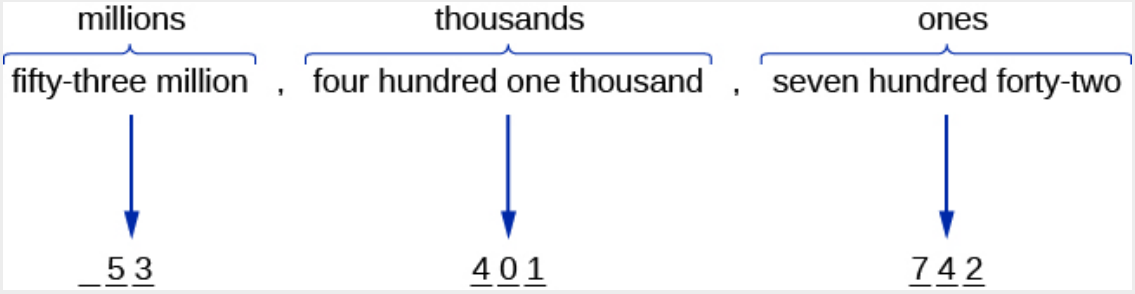
Example:
Exercise:

Problem: Write the following numbers using digits.

(a) fifty-three million, four hundred one thousand, seven hundred forty-two

Solution:
Solution

Write the digits in each period.



Put the numbers together, including the commas. The number is 53,401,742.

Note:
Exercise:

Problem: Write each number in standard form:

fifty-three million, eight hundred nine thousand, fifty-one.

Solution:

53,809,051

Note:
Exercise:

Problem: Write each number in standard form:

two billion, twenty-two million, seven hundred fourteen thousand, four hundred sixty-six.

Solution:

2,022,714,466

Example:
Exercise:

Problem:

A state budget was about $77 billion. Write the budget in standard form.

Solution:
Solution

Identify the periods. In this case, only two digits are given and they are in the billions period. To write the entire number, write zeros for all of the other periods.

The budget was about $77,000,000,000.

Note:
Exercise:

Problem: Write each number in standard form:

The closest distance from Earth to Mars is about 34 million miles.

Solution:

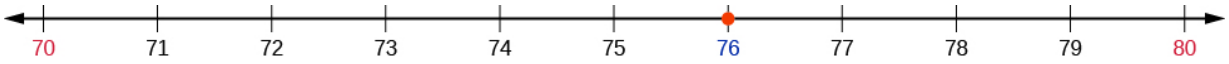
34,000,000 miles

Round Whole Numbers

In 2013, the U.S. Census Bureau reported the population of the state of New York as 19,651,127 people. It might be enough to say that the population is approximately 20 million. The word *approximately* means that 20 million is not the exact population, but is close to the exact value.

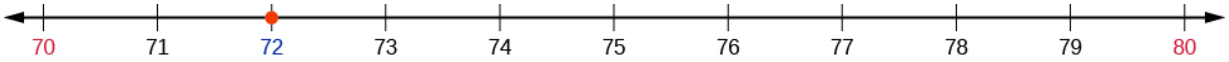
The process of approximating a number is called **rounding**. Numbers are rounded to a specific place value depending on how much accuracy is needed. Saying that the population of New York is approximately 20 million means we rounded to the millions place. The place value to which we round to depends on how we need to use the number.

Using the number line can help you visualize and understand the rounding process. Look at the number line in [link]. Suppose we want to round the number 76 to the nearest ten. Is 76 closer to 70 or 80 on the number line?



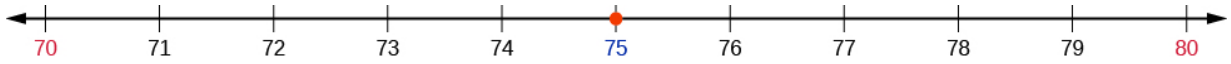
We can see that 76 is closer to 80 than to 70. So 76 rounded to the nearest ten is 80.

Now consider the number 72. Find 72 in [link].



We can see that 72 is closer to 70, so 72 rounded to the nearest ten is 70.

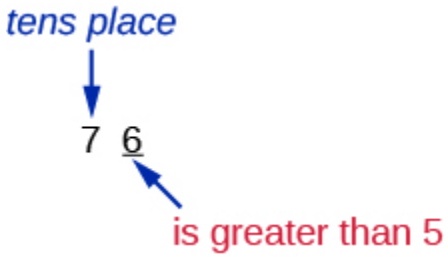
How do we round 75 to the nearest ten. Find 75 in [link].



The number 75 is exactly midway between 70 and 80.

So that everyone rounds the same way in cases like this, mathematicians have agreed to round to the higher number, 80. So, 75 rounded to the nearest ten is 80.

Now that we have looked at this process on the number line, we can introduce a more general procedure. To round a number to a specific place, look at the number to the right of that place. If the number is less than 5, round down. If it is greater than or equal to 5, round up.



The digit in the ones place is a 6. Because 6 is greater than or equal to 5, we increase the digit in the tens place by one. So the 7 in the tens place becomes an 8. Now, replace any digits to the right of the 8 with zeros. So, 76 rounds to 80.

Example:
Exercise:

Problem: Round 843 to the nearest ten.

Solution:
Solution

Locate the tens place.	*tens place* ↓ 843
Underline the digit to the right of the tens place.	843
Since 3 is less than 5, do not change the digit in the tens place.	843
Replace all digits to the right of the tens place with zeros.	840
Rounding 843 to the nearest ten gives 840.	

Note:
Exercise:

Problem: Round to the nearest ten: 157.

Solution:

160

Note:
Exercise:

Problem: Round to the nearest ten: 884.

Solution:

880

Example:
Exercise:

Problem: Round each number to the nearest hundred:

(a) 23,658

(b) 3,978

Solution:

(a) 23, 700 (b) 4,000

Note:
Exercise:

Problem: Round to the nearest hundred: 17,852.

Solution:

17,900

Example:
Exercise:

Problem: Round each number to the nearest thousand:

(a) 147,032

(b) 29,504

Solution:

(a) 147,000 (b) 30,000

We encourage you to go to Appendix B to take the Self Check for this section.

Note:
Access these online resources for additional instruction and practice with whole numbers.

- Determine Place Value
- Write a Whole Number in Digits from Words

Key Concepts

- Counting numbers begin with $1, 2, 3, 4$, and continue forever. Whole numbers are the counting numbers and zero: $0, 1, 2, 3, 4, \ldots$ See [link].
- Our number system uses a place value system, in which the value of a digit depends on its place in a number. The values can be represented by base-10 blocks. See [link].
- A place-value chart is a way to find the values of the digits in a number. See [link].
- Place values are used to write numbers in words. See [link].
- Numbers can be translated from words to digits. See [link].
- Rounding a number means to approximate it to a specific place value. See [link].

Practice Makes Perfect

Identify Counting Numbers and Whole Numbers In the following exercises, determine which of the following numbers are (a) counting numbers (b) whole numbers.

Exercise:

Problem: $0, \frac{2}{3}, 5, 8.1, 125$

Solution:

(a) 0, 5, 125 (b) 0, 5, 125

Exercise:

Problem: $0, \frac{7}{10}, 3, 20.5, 300$

Exercise:

Problem: $0, \frac{4}{9}, 3.9, 50, 221$

Solution:

(a) 50, 221 (b) 0, 50, 221

Exercise:

Problem: $0, \frac{3}{5}, 10, 303, 422.6$

Identify the Place Value of a Digit In the following exercises, find the place value of the given digits.

Exercise:

Problem: $579{,}601$

(a) 9

(b) 6

(c) 0

(d) 7

(e) 5

Solution:

(a) thousands (b) hundreds (c) tens (d) ten thousands (e) hundred thousands

Exercise:

Problem: 398,127

(a) 9

(b) 3

(c) 2

(d) 8

(e) 7

Exercise:

Problem: 56,804,379

(a) 8

(b) 6

(c) 4

(d) 7

(e) 0

Solution:

(a) hundred thousands (b) millions (c) thousands (d) tens (e) ten thousands

Exercise:

Problem: 78,320,465

(a) 8

(b) 4

(c) 2

(d) 6

(d) 7

Use Place Value to Name Whole Numbers In the following exercises, name each number in words.

Exercise:

Problem: 1,078

Solution:

One thousand, seventy-eight

Exercise:

Problem: 364,510

Solution:

Three hundred sixty-four thousand, five hundred ten

Exercise:

Problem: 5,846,103

Solution:

Five million, eight hundred forty-six thousand, one hundred three

Exercise:

Problem: 37,889,005

Solution:

Thirty seven million, eight hundred eighty-nine thousand, five

Exercise:

Problem: Seventy years is 613,200 hours.

Solution:

Six hundred thirteen thousand, two hundred

Exercise:

Problem: One year is 525,600 minutes.

Exercise:

Problem: The population of Chicago was 2,718,782.

Exercise:

Problem:

There are projected to be 23,867,000 college and university students in the US in five years.

Solution:

Twenty three million, eight hundred sixty-seven thousand

Exercise:

Problem:

The population of China is expected to reach $1,377,583,156$ in 2016.

Solution:

One billion, three hundred seventy-seven million, five hundred eighty-three thousand, one hundred fifty-six

Exercise:

Problem:

The population of India is estimated at $1,267,401,849$ as of July $1, 2014$.

Use Place Value to Write Whole Numbers In the following exercises, write each number as a whole number using digits.

Exercise:

Problem: four hundred twelve

Solution:

412

Exercise:

Problem: thirty-five thousand, nine hundred seventy-five

Solution:

35,975

Exercise:

Problem:

eleven million, forty-four thousand, one hundred sixty-seven

Solution:

11,044,167

Exercise:

Problem:

three billion, two hundred twenty-six million, five hundred twelve thousand, seventeen

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Solution:

3,226,512,017

Exercise:

Problem:

The population of the world was estimated to be seven billion, one hundred seventy-three million people.

Solution:

7,173,000,000

Exercise:

Problem:

Lake Tahoe has a capacity of thirty-nine trillion gallons of water.

Solution:

39,000,000,000,000

Round Whole Numbers In the following exercises, round to the indicated place value.

Exercise:

Problem: Round to the nearest ten: (a) 386 (b) 2,931

Solution:

(a) 390 (b) 2,930

Exercise:

Problem: Round to the nearest hundred: (a) 13,748 (b) 391,794

Solution:

(a) 13,700 (b) 391,800

Exercise:

Problem: Round to the nearest ten: (a) 1,492 (b) 1,497

Solution:

(a) 1490 (b) 1500

Exercise:

Problem: Round to the nearest hundred: (a) 63,994 (b) 63,949

Solution:

(a) 64,000 (b) 63,900

Exercise:

Problem: Round to the nearest thousand: (a) 163,584 (b) 163,246

Solution
(a) 164,000 (b) 163,000

Everyday Math

Exercise:

Problem:

Writing a Check Jorge bought a car for $24,493. He paid for the car with a check. Write the purchase price in words.

Solution:

Twenty four thousand, four hundred ninety-three dollars

Exercise:

Problem:

Remodeling a Kitchen Marissa's kitchen remodeling cost $18,549. Round the cost to the nearest:

(a) ten dollars

(b) hundred dollars

(c) thousand dollars

(d) ten-thousand dollars

Solution
(a) $18,550 (b) $18,600 (c) $19,000 (d) $20,000

Glossary

coordinate
a number paired with its point on a number line

counting numbers
: used for counting which start at 1 and continue without end, $\{1, 2, 3, 4, \ldots\}$, also known as natural numbers

ellipsis
: notation composed of three dots that indicate that numbers continue forever

estimate
: a number that is close to another number, but not exactly

natural numbers
: numbers used for counting which extend from 1 to infinity, $\{1, 2, 3, 4, \ldots\}$, also known as counting numbers

origin
: the point labeled zero on a number line

period
: group of three places values within a number, such as billions, millions, thousands, hundreds, tens, and ones

place value chart
: a method of summarizing information about the value of each digit in a number

place value system
: number system in which the value of a digit depends on its position, or place

rounding
: a process of estimating, or approximating a number

whole numbers
: counting numbers and zero, $\{0, 1, 2, 3, 4, \ldots\}$

Adding Whole Numbers
By the end of this section, you will be able to:

- Use addition notation
- Model addition of whole numbers
- Add whole numbers without models
- Translate word phrases to math notation
- Add whole numbers in applications

Use Addition Notation

A college student has a part-time job. Last week he worked 3 hours on Monday and 4 hours on Friday. To find the total number of hours he worked last week, he added 3 and 4.

The operation of addition combines numbers to get a **sum**. The notation we use to find the sum of 3 and 4 is:
Equation:

$$3 + 4$$

We read this as *three plus four* and the result is the sum of three and four. The numbers 3 and 4 are called the **addends**. A math statement that includes numbers and operations is called an **expression**.

Adding Whole Numbers

Note:
Identity Property of Addition
According to the Identity Property of Addition, the sum of any number a and 0 is the number.
Equation:

$$a + 0 = a$$
$$0 + a = a$$

Example:
Exercise:

Problem: Find each sum: $(\text{a})\ 0 + 11\ (\text{b})\ 42 + 0$.

Solution:

$0 + 11 = 11$

$42 + 0 = 42$

Look at the pairs of sums.
Equation:

$$\begin{aligned} 2 + 3 &= 5 \qquad & 3 + 2 &= 5 \\ 4 + 7 &= 11 \qquad & 7 + 4 &= 11 \\ 8 + 9 &= 17 \qquad & 9 + 8 &= 17 \end{aligned}$$

Notice that when the order of the addends is reversed, the sum does not change. This property is called the **Commutative Property of Addition**, which states that changing the order of the addends does not change their sum.

Note:
Commutative Property of Addition
According to the Commutative Property of Addition, changing the order of the addends a and b does not change their sum.
Equation:

$$a + b = b + a$$

Example:
Exercise:

Problem: Add: (a) $8 + 7$ (b) $7 + 8$.

Solution:
Solution

(a) Add. $8 + 7$
15

(b) Add. $7 + 8$
15

Example:
Exercise:

Problem: Add: $28 + 61$.

Solution:

To add numbers with more than one digit, it is often easier to write the numbers vertically in columns.

Write the numbers so the ones and tens digits line up vertically.

$$\begin{array}{r} 28 \\ +61 \\ \hline \end{array}$$

Then add the digits in each place value.

Add the ones: $8 + 1 = 9$

Add the tens: $2 + 6 = 8$

$$\begin{array}{r} 28 \\ +61 \\ \hline 89 \end{array}$$

Note:
Exercise:

Problem: Add: $25 + 74$.

Solution:

25 + 74 = 99

When the sum in a place value column is greater than 9, we carry over to the next column to the left. Carrying is the same as regrouping by exchanging. For example, 10 ones for 1 ten or 10 tens for 1 hundred.

Note:
Add whole numbers.
Step 1. Write the numbers so each place value lines up vertically.
Step 2. Add the digits in each place value. Work from right to left starting with the ones place. If a sum in a place value is more than 9, carry to the next place value.

Step 3. Continue adding each place value from right to left, adding each place value and carrying if needed.

Example:
Exercise:

Problem: Add: $43 + 69$.

Solution:

Write the numbers so the digits line up vertically.

$$\begin{array}{r} 43 \\ +69 \\ \hline \end{array}$$

Add the digits in each place.

Add the ones: $3 + 9 = 12$

Write the 2 in the ones place in the sum.

Add the 1 ten to the tens place.

$$\begin{array}{r} {\scriptstyle 1} \\ 43 \\ +69 \\ \hline 2 \end{array}$$

Now add the tens: $1 + 4 + 6 = 11$

Write the 11 in the sum.

$$\begin{array}{r} {\scriptstyle 1} \\ 43 \\ +69 \\ \hline 112 \end{array}$$

Note:
Exercise:

Problem: Add: $35 + 98$.

Solution:

35 + 98 = 133

Note:
Exercise:

Problem: Add: $456 + 376$.

Solution:

456 + 376 = 832

Note:
Exercise:

Problem: Add: $269 + 578$.

Solution:

269 + 578 = 847

Example:
Exercise:

Problem: Add: $1{,}683 + 479$.

Solution:

When the addends have different numbers of digits, be careful to line up the corresponding place values starting with the ones and moving toward the left. 1,683 + 479 = 2,162

Note:
Exercise:

Problem: Add: $4{,}597 + 685$.

Solution:

4,597 + 685 = 5,282

Example:
Exercise:

Problem: Add: $21{,}357 + 861 + 8{,}596$.

Solution:

This example had three addends. We can add any number of addends using the same process as long as we are careful to line up the place values correctly. Solution: 30,814

Note:
Exercise:

Problem: Add: $46{,}195 + 397 + 6{,}281$.

Solution:

46,195 + 397 + 6,281 = 52,873

Note:
Exercise:

Problem: Add: $53{,}762 + 196 + 7{,}458$.

Solution:

53,762 + 196 + 7,458 = 61,416

Translate Word Phrases to Math Notation

Earlier in this section, we translated math notation into words. Now we'll reverse the process. We'll translate word phrases into math notation. Some of the word phrases that indicate addition are listed in [link].

Operation	Words	Example	Expression

Operation	Words	Example	Expression
Addition	plus sum increased by more than total of added to	1 plus 2 the sum of 3 and 4 5 increased by 6 8 more than 7 the total of 9 and 5 6 added to 4	$1+2$ $3+4$ $5+6$ $7+8$ $9+5$ $4+6$

Example:
Exercise:

Problem: Translate and simplify: the sum of 19 and 23.

Solution:
Solution

The word *sum* tells us to add. The words *of* 19 *and* 23 tell us the addends.

Translate.	$19+23$
Add.	42

The sum of 19 and 23 is 42.

Note:
Exercise:

Problem: Translate and simplify: the sum of 17 and 26.

Solution:

Translate: 17 + 26; Simplify: 43

Example:
Exercise:

Problem: Translate and simplify: 28 increased by 31.

Solution:
Solution

The words *increased by* tell us to add. The numbers given are the addends.

28 increased by 31.

Translate.	$28 + 31$
Add.	59

So 28 increased by 31 is 59.

Note:
Exercise:

Problem: Translate and simplify: 29 increased by 76.

Solution:

Translate 29 + 76; Simplify 105

Add Whole Numbers in Applications

Now that we have practiced adding whole numbers, let's use what we've learned to solve real-world problems. We'll start by outlining a plan. First, we need to read the problem to determine what we are looking for. Then we write a word phrase that gives the information to find it. Next we translate the word phrase into math notation and then simplify. Finally, we write a sentence to answer the question.

Example:
Exercise:

Problem:

Hao earned grades of $87, 93, 68, 95,$ and 89 on the five tests of the semester. What is the total number of points he earned on the five tests?

Solution:

Write a phrase.	the sum of points on the tests
Translate to math notation.	$87 + 93 + 68 + 95 + 89$
Then we simplify by adding.	
Since there are several numbers, we will write them vertically.	$\begin{array}{r} {\scriptstyle 3} \\ 87 \\ 93 \\ 68 \\ 95 \\ +89 \\ \hline 432 \end{array}$
Write a sentence to answer the question.	Hao earned a total of 432 points.

Notice that we added *points*, so the sum is 432 *points*. It is important to include the appropriate units in all answers to applications problems.

Note:
Exercise:

Problem:

Mark is training for a bicycle race. Last week he rode 18 miles on Monday, 15 miles on Wednesday, 26 miles on Friday, 49 miles on Saturday, and 32 miles on Sunday. What is the total number of miles he rode last week?

Solution:

He rode 140 miles.

Note:
Exercise:

Problem:

Lincoln Middle School has three grades. The number of students in each grade is 230, 165, and 325. What is the total number of students?

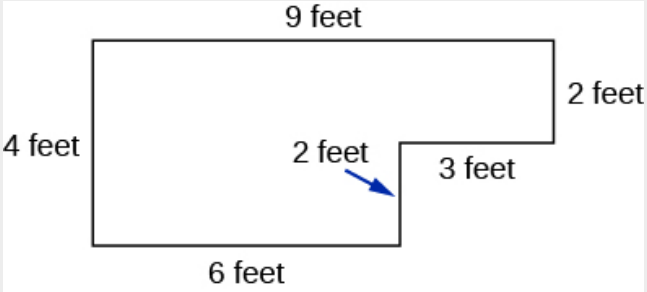
Solution:

The total number is 720 students.

Some application problems involve shapes. For example, a person might need to know the distance around a garden to put up a fence or around a picture to frame it. The **perimeter** is the distance around a geometric figure. The perimeter of a figure is the sum of the lengths of its sides.

Example:
Exercise:

Problem: Find the perimeter of the patio shown.

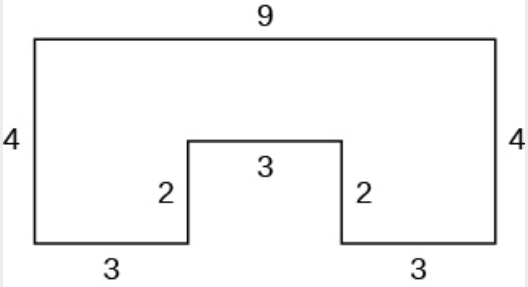


Solution:
Solution

We are asked to find the perimeter.	
Write a phrase.	the sum of the sides
Translate to math notation.	$4 + 6 + 2 + 3 + 2 + 9$
Simplify by adding.	26
Write a sentence to answer the question.	
We added feet, so the sum is 26 feet.	The perimeter of the patio is 26 feet.

Note:
Exercise:

Problem: Find the perimeter of each figure. All lengths are in inches.



Solution:

The perimeter is 30 inches.

Note:
Access these online resources for additional instruction and practice with adding whole numbers.

- Adding Whole Numbers

Key Concepts

- The Identity Property of Addition states that the sum of any number and 0 is the number. See [link].
- The Commutative Property of Addition states that changing the order of the addends does not change their sum. See [link].
- Numbers can be added horizontally or vertically. When they are written vertically, it is important to align digits in the same place value. See [link].
- When adding numbers with more than two-digits write the numbers vertically, align digits in the same place value, and add one place value at a time. See [link].
- Just as math notation can be translated into words, English word phrases can be translated into math notation. Key words and phrases, such as *sum*, *increased by*, *total*, and *sum* indicate addition. See [link].
- To solve applications first identify what is being asked, then write a phrase and translate it to math notation. Finally do the math and write a complete sentence to answer the question. See [link].

Practice Makes Perfect

Add Whole Numbers In the following exercises, fill in the missing values in each chart.

Exercise:

Problem:

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2		4	5	6	7		9
1	1	2	3	4			7	8	9	
2		3	4	5	6		8			11
3	3		5		7	8		10		12
4	4	5			8	9		11	12	
5	5	6	7	8			11		13	
6	6	7	8		10			13		15
7			9	10		12			15	16
8	8	9		11			14		16	
9	9	10	11		13	14			17	

Solution:

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Exercise:

Problem:

+	3	4	5	6	7	8	9
6							
7							
8							
9							

Solution:

+	3	4	5	6	7	8	9
6	9	10	11	12	13	14	15
7	10	11	12	13	14	15	16
8	11	12	13	14	15	16	17
9	12	13	14	15	16	17	18

In the following exercises, add.

Exercise:

Problem: $(\text{a})\ 0+13\ (\text{b})\ 13+0$

Solution:

(a) 13 (b) 13

Exercise:

Problem: $(\text{a})\ 8+3\ (\text{b})\ 3+8$

Solution:

(a) 11 (b) 11

Exercise:

Problem: $45+33$

Solution:

78

Exercise:

Problem: $71+28$

Solution:

99

Exercise:

Problem: $26+59$

Solution:

85

Exercise:

Problem: $64+78$

Solution:

142

Exercise:

Problem: $168 + 325$

Solution:

493

Exercise:

Problem: $584 + 277$

Solution:

861

Exercise:

Problem: $832 + 199$

Solution:

1,031

Exercise:

Problem: $6{,}358 + 492$

Solution:

6,850

Exercise:

Problem: $3{,}740 + 18{,}593$

Solution:

22,333

Exercise:

Problem: $485{,}012 + 619{,}848$

Solution:

1,104,860

Exercise:

Problem: $24{,}731 + 592 + 3{,}868$

Solution:

29,191

Exercise:

Problem: $8{,}015 + 76{,}946 + 16{,}570$

Solution:

101,531

Translate Word Phrases to Math Notation In the following exercises, translate each phrase into math notation and then simplify.

Exercise:

Problem: the sum of 13 and 18

Solution:

13 + 18 = 31

Exercise:

Problem: the sum of 90 and 65

Solution:

90 + 65 = 155

Exercise:

Problem: 33 increased by 49

Solution:

33 + 49 = 82

Exercise:

Problem: 250 more than 599

Solution:

250 + 599 = 849

Exercise:

Problem: the total of 628 and 77

Solution:

628 + 77 = 705

Exercise:

Problem: 1,482 added to 915

Solution:

915 + 1,482 = 2,397

Add Whole Numbers in Applications In the following exercises, solve the problem.

Exercise:

Problem:

Home remodeling Sophia remodeled her kitchen and bought a new range, microwave, and dishwasher. The range cost $1,100, the microwave cost $250, and the dishwasher cost $525. What was the total cost of these three appliances?

Solution:

The total cost was $1,875.

Exercise:

Problem:

Bike riding Ethan rode his bike 14 miles on Monday, 19 miles on Tuesday, 12 miles on Wednesday, 25 miles on Friday, and 68 miles on Saturday. What was the total number of miles Ethan rode?

Solution:

Ethan rode 138 miles.

Exercise:

Problem:

Apartment size Jackson lives in a 7 room apartment. The number of square feet in each room is 238, 120, 156, 196, 100, 132, and 225. What is the total number of square feet in all 7 rooms?

Solution:

The total square footage in the rooms is 1,167 square feet.

Exercise:

Problem:

Salary Last year Natalie's salary was $82,572. Two years ago, her salary was $79,316, and three years ago it was $75,298. What is the total amount of Natalie's salary for the past three years?

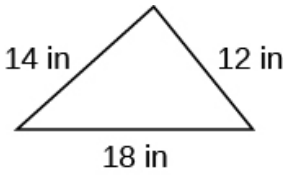
Solution:

Natalie's total salary is $237,186.

In the following exercises, find the perimeter of each figure.

Exercise:

Problem:

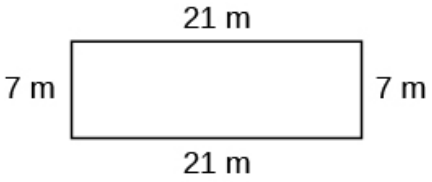


Solution:

The perimeter of the figure is 44 inches.

Exercise:

Problem:

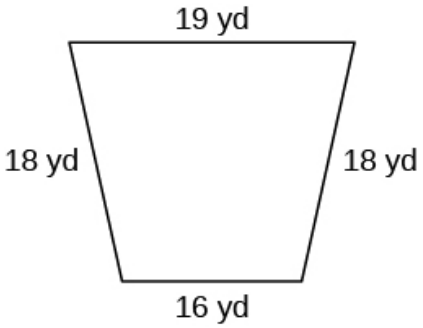


Solution:

The perimeter of the figure is 56 meters.

Exercise:

Problem:

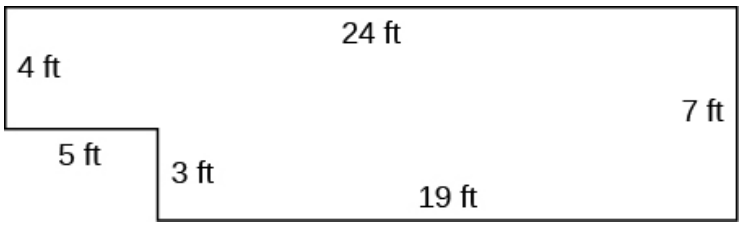


Solution:

The perimeter of the figure is 71 yards.

Exercise:

Problem:



Solution:

The perimeter of the figure is 62 feet.

Everyday Math

Exercise:

Problem:

Calories Paulette had a grilled chicken salad, ranch dressing, and a 16-ounce drink for lunch. On the restaurant's nutrition chart, she saw that each item had the following number of calories:

Grilled chicken salad – 320 calories
Ranch dressing – 170 calories
16-ounce drink – 150 calories

What was the total number of calories of Paulette’s lunch?

Solution:

The total number of calories was 640.

Exercise:

Problem:

Test scores A students needs a total of 400 points on five tests to pass a course. The student scored 82, 91, 75, 88, and 70. Did the student pass the course?

Solution:

Yes, he scored 406 points.

Glossary

addends
: numbers that are added together

additive identity
: number that does not change the sum when added to another number; zero

Commutative Property of Addition
: property that states that changing the order of the addends does not change their sum

equation
: math sentence that shows that two expressions are equal

expression
: math statement that includes numbers and operations

Identity Property of Addition
: states that the sum of any number a and 0 is the number

perimeter
: the distance around a geometric figure; the sum of the lengths of the sides

sum
: the result of adding numbers

Subtracting Whole Numbers
By the end of this section, you will be able to:

- Use subtraction notation
- Model subtraction of whole numbers
- Subtract whole numbers
- Translate word phrases to math notation
- Subtract whole numbers in applications

Use Subtraction Notation

Suppose there are seven bananas in a bowl. Elana uses three of them to make a smoothie. How many bananas are left in the bowl? To answer the question, we subtract three from seven. When we subtract, we take one number away from another to find the **difference**. The notation we use to subtract 3 from 7 is
Equation:

$$7 - 3$$

We read $7 - 3$ as *seven minus three* and the result is *the difference of seven and three*.

Subtract Whole Numbers

Addition and subtraction are **inverse operations**. Addition undoes subtraction, and subtraction undoes addition.

We know $7 - 3 = 4$ because $4 + 3 = 7$. Knowing all the addition number facts will help with subtraction. Then we can check subtraction by adding. In the examples above, our subtractions can be checked by addition.
Equation:

$$\begin{array}{ccc} 7 - 3 = 4 & \text{because} & 4 + 3 = 7 \\ 13 - 8 = 5 & \text{because} & 5 + 8 = 13 \\ 43 - 26 = 17 & \text{because} & 17 + 26 = 43 \end{array}$$

Note:
Exercise:

Problem: Subtract and then check by adding:

$7 - 0$

Solution:

$7 - 0 = 7$; $7 + 0 = 7$

To subtract numbers with more than one digit, it is usually easier to write the numbers vertically in columns just as we did for addition. Align the digits by place value, and then subtract each column starting with the ones and then working to the left.

Example:
Exercise:

Problem: Subtract and then check by adding: $89 - 61$.

Solution:
Solution

Write the numbers so the ones and tens digits line up vertically.	$\begin{array}{r} 89 \\ -61 \\ \hline \end{array}$
Subtract the digits in each place value.	
Subtract the ones: $9 - 1 = 8$	$\begin{array}{r} 89 \\ -61 \\ \hline 28 \end{array}$
Subtract the tens: $8 - 6 = 2$	$\begin{array}{r} 28 \\ +61 \\ \hline 89 \end{array}$
Check using addition.	

Our answer is correct.

Note:
Exercise:

Problem: Subtract and then check by adding: $86 - 54$.

Solution:

$86 - 54 = 32$ because $54 + 32 = 86$

Note:
Exercise:

Problem: Subtract and then check by adding: $99 - 74$.

Solution:

$99 - 74 = 25$ because $74 + 25 = 99$

Note:
Find the difference of whole numbers.
Step 1. Write the numbers so each place value lines up vertically.
Step 2. Subtract the digits in each place value. Work from right to left starting with the ones place. If the digit on top is less than the digit below, borrow as needed.
Step 3. Continue subtracting each place value from right to left, borrowing if needed.
Step 4. Check by adding.

Example:
Exercise:

Problem: Subtract: $43 - 26$.

Solution:
Solution

Write the numbers so each place value lines up vertically.	$\begin{array}{r} 43 \\ -26 \\ \hline \end{array}$
Subtract the ones. We cannot subtract 6 from 3, so we borrow 1 ten. This makes 3 tens and 13 ones. We write these numbers above each place and cross out the original digits.	$\begin{array}{r} \cancel{4}^{3}\,\cancel{3}^{13} \\ -26 \\ \hline \end{array}$
Now we can subtract the ones. $13 - 6 = 7$. We write the 7 in the ones place in the difference.	$\begin{array}{r} \cancel{4}^{3}\,\cancel{3}^{13} \\ -26 \\ \hline 7 \end{array}$
Now we subtract the tens. $3 - 2 = 1$. We write the 1 in the tens place in the difference.	$\begin{array}{r} \cancel{4}^{3}\,\cancel{3}^{13} \\ -26 \\ \hline 17 \end{array}$
Check by adding.	$\begin{array}{r} 17 \\ +26 \\ \hline 43\checkmark \end{array}$
Our answer is correct.	

Note:
Exercise:

Problem: Subtract and then check by adding: $93 - 58$.

Solution:

93 − 58 = 35 because 58 + 35 = 93

Example:
Exercise:

Problem: Subtract and then check by adding: $207 - 64$.

Solution:
Solution

Write the numbers so each place value lines up vertically.	$\begin{array}{r} 207 \\ -\ 64 \\ \hline \end{array}$
Subtract the ones. $7 - 4 = 3$. Write the 3 in the ones place in the difference.	$\begin{array}{r} 207 \\ -\ 64 \\ \hline 3 \end{array}$
Subtract the tens. We cannot subtract 6 from 0, so we borrow 1 hundred and add 10 tens to the 0 tens we had. This makes a total of 10 tens. We write 10 above the tens place and cross out the 0. Then we cross out the 2 in the hundreds place and write 1 above it.	$\begin{array}{r} {\scriptstyle 1\ \ 10} \\ \cancel{2}\ \cancel{0}\,7 \\ -\ 64 \\ \hline 3 \end{array}$
Now we can subtract the tens. $10 - 6 = 4$. Write the 4 in the tens place in the difference.	$\begin{array}{r} {\scriptstyle 1\ \ 10} \\ \cancel{2}\ \cancel{0}\,7 \\ -\ 64 \\ \hline 43 \end{array}$
Finally, subtract the hundreds. There is no digit in the hundreds place in the bottom number so we can imagine a 0 in that place. Since $1 - 0 = 1$, we write 1 in the hundreds place in the difference.	$\begin{array}{r} {\scriptstyle 1\ \ 10} \\ \cancel{2}\ \cancel{0}\,7 \\ -\ 064 \\ \hline 143 \end{array}$
Check by adding.	$\begin{array}{r} {\scriptstyle 1} \\ 143 \\ +\ 64 \\ \hline 207\checkmark \end{array}$
Our answer is correct.	

Note:
Exercise:

Problem: Subtract and then check by adding: $318 - 75$.

Solution:

$318 - 75 = 243$ because $243 + 75 = 318$

Example:
Exercise:

Problem: Subtract and then check by adding: $910 - 586$.

Solution:
Solution

Write the numbers so each place value line up vertically.	$\begin{array}{r} 910 \\ -586 \\ \hline \end{array}$
Subtract the ones. We cannot subtract 6 from 0, so we borrow 1 ten and add 10 ones to the 10 ones we had. This makes 10 ones. We write a 0 above the tens place and cross out the 1. We write the 10 above the ones place and cross out the 0. Now we can subtract the ones. $10 - 6 = 4$.	$\begin{array}{r} {}^{0}\cancel{9}\,{}^{10}\cancel{1}\,{}^{10}\cancel{0} \\ -586 \\ \hline \end{array}$
Write the 4 in the ones place of the difference.	$\begin{array}{r} {}^{0}\cancel{9}\,{}^{10}\cancel{1}\,{}^{10}\cancel{0} \\ -586 \\ \hline 4 \end{array}$
Subtract the tens. We cannot subtract 8 from 0, so we borrow 1 hundred and add 10 tens to the 0 tens we had, which gives us 10 tens. Write 8 above the hundreds place and cross out the 9. Write 10 above the tens place. Now we can subtract the tens. $10 - 8 = 2$.	$\begin{array}{r} {}^{8}\cancel{9}\,{}^{10}\cancel{1}\,{}^{10}\cancel{0} \\ -586 \\ \hline 4 \end{array}$
Write 2 in the tens place in the difference.	$\begin{array}{r} {}^{8}\cancel{9}\,{}^{10}\cancel{1}\,{}^{10}\cancel{0} \\ -586 \\ \hline 24 \end{array}$
Subtract the hundreds place. $8 - 5 = 3$.	
Write the 3 in the hundreds place in the difference.	$\begin{array}{r} {}^{8}\cancel{9}\,{}^{10}\cancel{1}\,{}^{10}\cancel{0} \\ -586 \\ \hline 324 \end{array}$
Check by adding.	$\begin{array}{r} {}^{1}3\,{}^{1}2\,4 \\ +586 \\ \hline 910\checkmark \end{array}$
Our answer is correct.	

Note:

Exercise:

Problem: Subtract and then check by adding: $832 - 376$.

Solution:

832 − 376 = 456 because 456 + 376 = 832

Note:
Exercise:

Problem: Subtract and then check by adding: $847 - 578$.

Solution:

847 − 578 = 269 because 269 + 578 = 847

Note:
Exercise:

Problem: Subtract and then check by adding: $4{,}585 - 697$.

Solution:

4,585 − 697 = 3,888 because 3,888 + 697 = 4,585

Note:
Exercise:

Problem: Subtract and then check by adding: $5{,}637 - 899$.

Solution:

5,637 − 899 = 4,738 because 4,738 + 899 = 5,637

Translate Word Phrases to Math Notation

As with addition, word phrases can tell us to operate on two numbers using subtraction. To translate from a word phrase to math notation, we look for key words that indicate subtraction. Some of the words that indicate subtraction are listed in [link].

Operation	Word Phrase	Example	Expression
Subtraction	minus difference decreased by less than subtracted from	5 minus 1 the difference of 9 and 4 7 decreased by 3 5 less than 8 1 subtracted from 6	$5-1$ $9-4$ $7-3$ $8-5$ $6-1$

Example:
Exercise:

Problem: Translate and then simplify:

(a) the difference of 13 and 8

(b) subtract 24 from 43

Solution:
Solution

(a) The word *difference* tells us to subtract the two numbers. The numbers stay in the same order as in the phrase.

	the difference of 13 and 8
Translate.	$13-8$
Simplify.	5

(b) The words *subtract from* tells us to take one number away from the other. We must be careful to get the order correct.

	subtract 24 from 43
Translate.	$43-24$
Simplify.	19

Note:
Exercise:

Problem: Translate and simplify:

(a) the difference of 14 and 9 (b) subtract 21 from 37

Solution:

(a) 14 − 9 = 5 (b) 37 − 21 = 16

Note:
Exercise:

Problem: Translate and simplify:

(a) 11 decreased by 6 (b) 18 less than 67

Solution:

(a) 11 − 6 = 5 (b) 67 − 18 = 49

Subtract Whole Numbers in Applications

To solve applications with subtraction, we will use the same plan that we used with addition. First, we need to determine what we are asked to find. Then we write a phrase that gives the information to find it. We translate the phrase into math notation and then simplify to get the answer. Finally, we write a sentence to answer the question, using the appropriate units.

Example:
Exercise:

Problem:

The temperature in Chicago one morning was 73 degrees Fahrenheit. A cold front arrived and by noon the temperature was 27 degrees Fahrenheit. What was the difference between the temperature in the morning and the temperature at noon?

Solution:
Solution

We are asked to find the difference between the morning temperature and the noon temperature.

Write a phrase.	the difference of 73 and 27
Translate to math notation.	
Difference tells us to subtract.	$73 - 27$
Then we do the subtraction.	$\begin{array}{r} {}^{6}\cancel{7}\,{}^{13}\cancel{3} \\ -2\,7 \\ \hline 4\,6 \end{array}$
Write a sentence to answer the question.	The difference in temperatures was 46 degrees Fahrenheit.

Note:
Exercise:

Problem:

The high temperature on June 1^{st} in Boston was 77 degrees Fahrenheit, and the low temperature was 58 degrees Fahrenheit. What was the difference between the high and low temperatures?

Solution:

The difference is 19 degrees Fahrenheit.

Example:
Exercise:

Problem:

A washing machine is on sale for $399. Its regular price is $588. What is the difference between the regular price and the sale price?

Solution:
Solution

We are asked to find the difference between the regular price and the sale price.

Write a phrase.	the difference between 588 and 399
Translate to math notation.	$588 - 399$
Subtract.	$\begin{array}{r} {}^{4}\cancel{5}\,{}^{17}\cancel{8}\,{}^{18}\cancel{8} \\ -\,3\,9\,9 \\ \hline 1\,8\,9 \end{array}$
Write a sentence to answer the question.	The difference between the regular price and the sale price is $189.

Note:
Exercise:

Problem:

A television set is on sale for $499. Its regular price is $648. What is the difference between the regular price and the sale price?

Solution:

The difference is $149.

We encourage you to go to Appendix B to take the Self Check for this section.

Note:
Access these online resources for additional instruction and practice with subtracting whole numbers.

- Subtract Whole Numbers

Key Concepts

- Subtraction and addition are inverse operations. Addition can be used to check the result found by subtraction. See [link].
- When numbers to be subtracted are written vertically, it is important to align digits in the same place value. See [link].
- To subtract numbers with more than two-digits write the numbers vertically, aligning digits in the same place value, and subtract one place value at a time. See [link].
- Just as math notation can be translated into words, word phrases can be translated into math notation. Some key words and phrases that mean subtraction are *minus*, *difference*, *decreased*

by, and *subtracted from*. See [link].

Practice Makes Perfect

Use Subtraction Notation For the following exercises, translate from math notation to words.
Exercise:

Problem: $15 - 9$

Solution:

fifteen minus nine; the difference of fifteen and nine

Subtract Whole Numbers For the following exercises, subtract and then check by adding.
Exercise:

Problem: $9 - 4$

Solution:

5

Exercise:

Problem: $8 - 0$

Solution:

8

Exercise:

Problem: $38 - 16$

Solution:

22

Exercise:

Problem: $85 - 52$

Solution:

33

Exercise:

Problem: $493 - 370$

Solution:

123

Exercise:

Problem: $5{,}946 - 4{,}625$

Solution:

1,321

Exercise:

Problem: $75 - 47$

Solution:

28

Exercise:

Problem: $461 - 239$

Solution:

222

Exercise:

Problem: $525 - 179$

Solution:

346

Exercise:

Problem: $6{,}318 - 2{,}799$

Solution:

3,519

Exercise:

Problem: $2{,}150 - 964$

Solution:

1,186

Exercise:

Problem: $43{,}650 - 8{,}982$

Solution:

34,668

Translate Word Phrases to Algebraic Expressions For the following exercises, translate and simplify.

Exercise:

Problem: The difference of 10 and 3

Solution:

$10 - 3$; 7

Exercise:

Problem: The difference of 15 and 4

Solution:

$15 - 4$; 11

Exercise:

Problem: Subtract 28 from 75

Solution:

$75 - 28$; 47

Exercise:

Problem: 45 decreased by 20

Solution:

$45 - 20$; 25

Exercise:

Problem: 92 decreased by 67

Solution:

92 − 67; 25

Exercise:

Problem: 12 less than 16

Solution:

16 − 12; 4

Exercise:

Problem: 38 less than 61

Solution:

61 − 38; 23

Mixed Practice For the following exercises, simplify.

Exercise:

Problem: $256 - 184$

Solution:

72

Exercise:

Problem: $719 + 341$

Solution:

1,060

For the following exercises, translate and simplify.

Exercise:

Problem: Seventy more than thirty-five

Solution:

75 + 35; 110

Exercise:

Problem: Sixty more than ninety-three

Exercise:

Problem: 13 less than 41

Solution:

41 − 13; 28

Exercise:

Problem: 28 less than 36

Exercise:

Problem: The difference of 100 and 76

Solution:

100 − 76; 24

Exercise:

Problem: The difference of 1,000 and 945

Subtract Whole Numbers in Applications For the following exercises, solve.

Exercise:

Problem:

Temperature The high temperature on June 2 in Las Vegas was 80 degrees and the low temperature was 63 degrees. What was the difference between the high and low temperatures?

Solution:

The difference between the high and low temperature was 17 degrees

Exercise:

Problem:

Class size Olivia’s third grade class has 35 children. Last year, her second grade class had 22 children. What is the difference between the number of children in Olivia’s third grade class and her second grade class?

Solution:

The difference between the third grade and second grade was 13 children.

Exercise:

Problem:

Shopping A mountain bike is on sale for $399. Its regular price is $650. What is the difference between the regular price and the sale price?

Solution:

The difference between the regular price and sale price is $251.

Exercise:

Problem:

Savings John wants to buy a laptop that costs $840. He has $685 in his savings account. How much more does he need to save in order to buy the laptop?

Solution:

John needs to save $155 more.

Everyday Math

Exercise:

Problem:

Road trip Noah was driving from Philadelphia to Cincinnati, a distance of 502 miles. He drove 115 miles, stopped for gas, and then drove another 230 miles before lunch. How many more miles did he have to travel?

Solution:

157 miles

Writing Exercises

Exercise:

Problem: Explain how subtraction and addition are related.

Solution:

Answers may vary.

Exercise:

Problem: How does knowing addition facts help you to subtract numbers?

Glossary

difference
: result obtained when subtracting one number from another

inverse operations
: operations that undo each other, such as addition and subtraction

Multiplying Whole Numbers
By the end of this section, you will be able to:

- Use multiplication notation
- Model multiplication of whole numbers
- Multiply whole numbers
- Translate word phrases to math notation
- Multiply whole numbers in applications

Use Multiplication Notation

Suppose you were asked to count all these pennies shown in [link].

Would you count the pennies individually? Or would you count the number of pennies in each row and add that number 3 times.
Equation:

$$8 + 8 + 8$$

Multiplication is a way to represent repeated addition. So instead of adding 8 three times, we could write a multiplication expression.
Equation:

$$3 \times 8$$

We call each number being multiplied a **factor** and the result the **product**. We read 3×8 as *three times eight*, and the result as *the product of three and eight*.

There are several symbols that represent multiplication. These include the symbol $\times$ as well as the dot, $\cdot$, and parentheses $( )$.

Note:
Operation Symbols for Multiplication
To describe multiplication, we can use symbols and words.

Operation	Notation	Expression	Read as	Result
Multiplication	$\times$ $\cdot$ $( )$	3×8 $3 \cdot 8$ $3(8)$	three times eight	the product of 3 and 8

Note:
Exercise:

Problem: Translate from math notation to words:

(a) 8×7 (b) $18 \cdot 11$

Solution:

(a) eight times seven ; the product of eight and seven (b) eighteen times eleven ; the product of eighteen and eleven

Note:
Exercise:

Problem: Translate from math notation to words:

(a) $(13)(7)$ (b) $5(16)$

Solution:

(a) thirteen times seven ; the product of thirteen and seven (b) five times sixteen; the product of five and sixteen

Multiply Whole Numbers

You will need to know all the one digit multiplication facts. Make sure you know them fluently before proceeding in this section.

[link] shows the multiplication facts. Each box shows the product of the number down the left column and the number across the top row. If you are unsure about a product, model it. It is important that you memorize any number facts you do not already know so you will be ready to multiply larger numbers.

$\times$	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

What happens when you multiply a number by zero? You can see that the product of any number and zero is zero. This is called the **Multiplication Property of Zero**.

Note:
Multiplication Property of Zero
According to the Multiplication Property of Zero, the product of any number and 0 is 0.
Equation:

$$a \cdot 0 = 0$$
$$0 \cdot a = 0$$

Example:
Exercise:

Problem: Multiply: (a) $0 \cdot 11$ (b) $(42)0$.

Solution:

(a) The product of any number and zero is zero.

$$0 \cdot 11$$
$$0$$

(b) Multiplying by zero results in zero.

$$(42)0$$
$$0$$

What happens when you multiply a number by one? Multiplying a number by one does not change its value. We call this fact the **Identity Property of Multiplication**, and 1 is called the **multiplicative identity**.

Note:
Identity Property of Multiplication
According to the Identity Property of Multiplication, the product of any number and 1 is the number.
Equation:

$$1 \cdot a = a$$
$$a \cdot 1 = a$$

Example:
Exercise:

Problem: Multiply: (a) $(11)1$ (b) $1 \cdot 42$.

Solution:
Solution

	$(11)1$
(a) The product of any number and one is the number.	11
	$1 \cdot 42$
(b) Multiplying by 1 does not change the value.	42

Note:
Exercise:

Problem: Find each product:

(a) $(19)1$ (b) $1 \cdot 39$

Solution:

(a) 19 (b) 39

Earlier in this chapter, we learned that the Commutative Property of Addition states that changing the order of addition does not change the sum. We saw that $8 + 9 = 17$ is the same as $9 + 8 = 17$.

Is this also true for multiplication? Let's look at a few pairs of factors.
Equation:

$$4 \cdot 7 = 28 \qquad 7 \cdot 4 = 28$$

Equation:

$$9 \cdot 7 = 63 \qquad 7 \cdot 9 = 63$$

Equation:

$$8 \cdot 9 = 72 \qquad 9 \cdot 8 = 72$$

When the order of the factors is reversed, the product does not change. This is called the **Commutative Property of Multiplication**.

Note:
Commutative Property of Multiplication
According to the Commutative Property of Multiplication, changing the order of the factors does not change their product.
Equation:

$$a \cdot b = b \cdot a$$

Example:
Exercise:

Problem: Multiply: (a) $8 \cdot 7$ (b) $7 \cdot 8$.

Solution:
Solution

	$8 \cdot 7$
(a) Multiply.	56
	$7 \cdot 8$
(b) Multiply.	56

Note:
Exercise:

Problem: Multiply: (a) $9 \cdot 6$ (b) $6 \cdot 9$.

Solution:

54 and 54; both are the same.

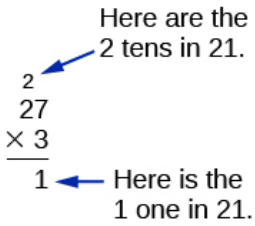
To multiply numbers with more than one digit, it is usually easier to write the numbers vertically in columns just as we did for addition and subtraction.
Equation:

$$\begin{array}{r} 27 \\ \times 3 \\ \hline \end{array}$$

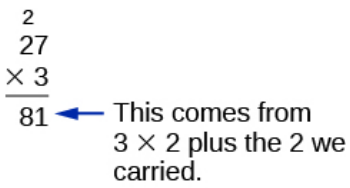
We start by multiplying 3 by 7.
Equation:

$$3 \times 7 = 21$$

We write the 1 in the ones place of the product. We carry the 2 tens by writing 2 above the tens place.



Then we multiply the 3 by the 2, and add the 2 above the tens place to the product. So $3 \times 2 = 6$, and $6 + 2 = 8$. Write the 8 in the tens place of the product.



The product is 81.

When we multiply two numbers with a different number of digits, it's usually easier to write the smaller number on the bottom. You could write it the other way, too, but this way is easier to work with.

Example:
Exercise:

Problem: Multiply: $15 \cdot 4$.

Solution:

Write the numbers so the digits 5 and 4 line up vertically.	$\begin{array}{r} 15 \\ \times\ 4 \\ \hline \end{array}$
Multiply 4 by the digit in the ones place of 15. $4 \cdot 5 = 20$.	
Write 0 in the ones place of the product and carry the 2 tens.	$\begin{array}{r} \overset{2}{15} \\ \times\ 4 \\ \hline 0 \end{array}$
Multiply 4 by the digit in the tens place of 15. $4 \cdot 1 = 4$. Add the 2 tens we carried. $4 + 2 = 6$.	
Write the 6 in the tens place of the product.	$\begin{array}{r} \overset{2}{15} \\ \times\ 4 \\ \hline 60 \end{array}$

Note:
Exercise:

Problem: Multiply: $64 \cdot 8$.

Solution:

518

Note:
Exercise:

Problem: Multiply: $57 \cdot 6$.

Solution:

342

Example:
Exercise:

Problem: Multiply: $286 \cdot 5$.

Solution:

Write the numbers so the digits 5 and 6 line up vertically.	$\begin{array}{r} 286 \\ \times\ \ 5 \\ \hline \end{array}$
Multiply 5 by the digit in the ones place of $286.\ 5 \cdot 6 = 30.$	$\begin{array}{r} {\scriptstyle 3} \\ 286 \\ \times\ \ 5 \\ \hline \end{array}$
Write the 0 in the ones place of the product and carry the 3 to the tens place.	$\begin{array}{r} 0 \end{array}$
Multiply 5 by the digit in the tens place of $286.\ 5 \cdot 8 = 40.$	$\begin{array}{r} {\scriptstyle 4\ 3} \\ 286 \\ \times\ \ 5 \\ \hline 30 \end{array}$
Add the 3 tens we carried to get $40 + 3 = 43.$	
Write the 3 in the tens place of the product and carry the 4 to the hundreds place.	
Multiply 5 by the digit in the hundreds place of $286.\ 5 \cdot 2 = 10.$	$\begin{array}{r} {\scriptstyle 4\ 3} \\ 286 \\ \times\ \ 5 \\ \hline \end{array}$
Add the 4 hundreds we carried to get $10 + 4 = 14.$	$\begin{array}{r} 1{,}430 \end{array}$
Write the 4 in the hundreds place of the product and the 1 to the thousands place.	

Note:
Exercise:

Problem: Multiply: $347 \cdot 5$.

Solution:

1,735

Note:
Exercise:

Problem: Multiply: $462 \cdot 7$.

Solution:

3,234

When we multiply by a number with two or more digits, we multiply by each of the digits separately, working from right to left. Each separate product of the digits is called a **partial product**. When we write

partial products, we must make sure to line up the place values.

Note:
Multiply two whole numbers to find the product.
Step 1. Write the numbers so each place value lines up vertically.
Step 2. Multiply the digits in each place value.

- Work from right to left, starting with the ones place in the bottom number.
    - Multiply the bottom number by the ones digit in the top number, then by the tens digit, and so on.
    - If a product in a place value is more than 9, carry to the next place value.
    - Write the partial products, lining up the digits in the place values with the numbers above.
- Repeat for the tens place in the bottom number, the hundreds place, and so on.
- Insert a zero as a placeholder with each additional partial product.

Step 3. Add the partial products.

Example:
Exercise:

Problem: Multiply: $62(87)$.

Solution:
Solution

Write the numbers so each place lines up vertically.

$$\begin{array}{r} 62 \\ \times\ 87 \\ \hline \end{array}$$

Start by multiplying 7 by 62.
Multiply 7 by the digit in the ones place of 67. $7 \cdot 2 = 14$.
Write the 4 in the ones place of the product and carry the 1 to the tens place.

$$\begin{array}{r} {\scriptstyle 1} \\ 62 \\ \times\ 87 \\ \hline 4 \end{array}$$

Multiply 7 by the digit in the tens place of 62. $7 \cdot 6 = 42$.
Add the 1 ten we carried. $42 + 1 = 43$.
Write the 3 in the tens place of the product and the 4 in the hundreds place.

$$\begin{array}{r} {\scriptstyle 1} \\ 62 \\ \times\ 87 \\ \hline 434 \end{array}$$

The first partial product is 434.

Now, write a 0 under the 4 in the ones place of the next partial product as a placeholder since we now multiply the digit in the tens place of 87 by 62.
Multiply 8 by the digit in the ones place of 62. $8 \cdot 2 = 16$.
Write the 6 in the next place of the product, which is the tens place.
Carry the 1 to the tens place.

$$\begin{array}{r} {\scriptstyle 1} \\ 62 \\ \times\ 87 \\ \hline 434 \\ 60 \end{array}$$

Multiply 8 by 6, the digit in the tens place of 62, then add the 1 ten we carried to get 49.
Write the 9 in the hundreds place of the product and the 4 in the thousands place.

$$\begin{array}{r} {\scriptstyle 1} \\ 62 \\ \times\ 87 \\ \hline 434 \\ 4960 \end{array}$$

The second partial product is 4960
Add the partial products.

$$\begin{array}{r} {\scriptstyle 1} \\ 62 \\ \times\ 87 \\ \hline 434 \\ 4960 \\ \hline 5394 \end{array}$$

The product is 5,394.

Note:
Exercise:

Problem: Multiply: $43(78)$.

Solution:

3,354

Note:
Exercise:

Problem: Multiply: $64(59)$.

Solution:

3,776

Example:
Exercise:

Problem: Multiply: (a) $47 \cdot 10$ (b) $47 \cdot 100$.

Solution:
Solution

(a) Multiply 0 by 47 to get the first partial product, which is 0. Then multiply 1 by 47 to get the second partial product. Be sure to insert a 0 in the ones place of the second partial product as a placeholder. Then add the partial products.

$$\begin{array}{r} 47 \\ \times 10 \\ \hline 00 \\ 470 \\ \hline 470 \end{array}$$

(b) Multiply 0 by 47 to get the first partial product, which is 0. Multiply 0 by 47 again to get the second partial product, which is 0. Be sure to insert a 0 in the ones place as a placeholder. Then multiply 1 by 47 to get the third partial product. Be sure to insert two 0's to the ones and tens places as placeholders. Each partial product has one more zero as a placeholder. Then add the partial products.

$$\begin{array}{r} 47 \\ \times 100 \\ \hline 00 \\ 000 \\ 4700 \\ \hline 4,700 \end{array}$$

When we multiplied 47 times 10, the product was 470. Notice that 10 has one zero, and we put one zero after 47 to get the product. When we multiplied 47 times 100, the product was 4,700. Notice that 100 has two zeros and we put two zeros after 47 to get the product.

Do you see the pattern? If we multiplied 47 times 10,000, which has four zeros, we would put four zeros after 47 to get the product 470,000.

Note:
Exercise:

Problem: Multiply: (a) $54 \cdot 10$ (b) $54 \cdot 100$.

Solution:

(a) 540 (b) 5,400

Note:
Exercise:

Problem: Multiply: (a) $75 \cdot 10$ (b) $75 \cdot 100$.

Solution:

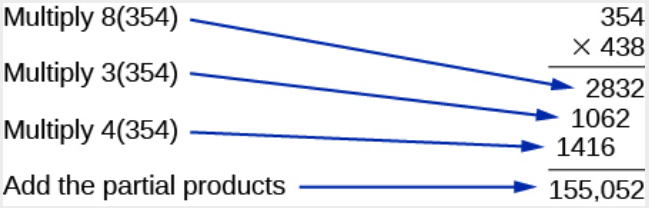
(a) 750 (b) 7,500

Example:
Exercise:

Problem: Multiply: $(354)(438)$.

Solution:
Solution

There are three digits in the factors so there will be 3 partial products. We do not have to write the 0 as a placeholder as long as we write each partial product in the correct place.



Note:
Exercise:

Problem: Multiply: $(265)(483)$.

Solution:

127,995

Note:
Exercise:

Problem: Multiply: $(823)(794)$.

Solution:

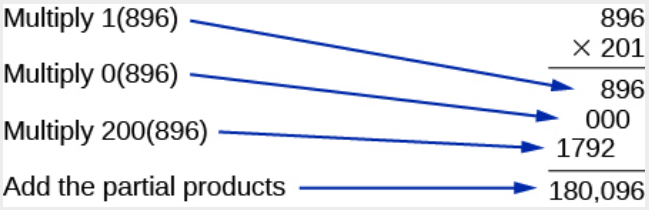
653,462

Example:
Exercise:

Problem: Multiply: $(896)201$.

Solution:
Solution

There should be 3 partial products. The second partial product will be the result of multiplying 896 by 0.



Notice that the second partial product of all zeros doesn't really affect the result. We can place a zero as a placeholder in the tens place and then proceed directly to multiplying by the 2 in the hundreds place, as shown.

Multiply by 10, but insert only one zero as a placeholder in the tens place. Multiply by 200, putting the 2 from the 12. $2 \cdot 6 = 12$ in the hundreds place.

$$\begin{array}{r} 896 \\ \times 201 \\ \hline 896 \\ 17920 \\ \hline 180{,}096 \end{array}$$

Note:
Exercise:

Problem: Multiply: $(718)509$.

Solution:

365,462

Note:
Exercise:

Problem: Multiply: $(627)804$.

Solution:

504,108

When there are three or more factors, we multiply the first two and then multiply their product by the next factor. For example:
Equation:

$$\begin{array}{ll} \text{to multiply} & 8 \cdot 3 \cdot 2 \\ \text{first multiply } 8 \cdot 3 & 24 \cdot 2 \\ \text{then multiply } 24 \cdot 2. & 48 \end{array}$$

Translate Word Phrases to Math Notation

Earlier in this section, we translated math notation into words. Now we'll reverse the process and translate word phrases into math notation. Some of the words that indicate multiplication are given in [link].

Operation	Word Phrase	Example	Expression
Multiplication	times product twice	3 times 8 the product of 3 and 8 twice 4	$3 \times 8, 3 \cdot 8, (3)(8), (3)8,$ or $3(8)$ $2 \cdot 4$

Example:
Exercise:

Problem: Translate and simplify: the product of 12 and 27.

Solution:

The word *product* tells us to multiply. The words *of* 12 *and* 27 tell us the two factors.

	the product of 12 and 27
Translate.	$12 \cdot 27$
Multiply.	324

Note:
Exercise:

Problem: Translate and simplify the product of 13 and 28.

Solution:

$13 \cdot 28$; 364

Note:
Exercise:

Problem: Translate and simplify the product of 47 and 14.

Solution:

$47 \cdot 14$; 658

Example:
Exercise:

Problem: Translate and simplify: twice two hundred eleven.

Solution:
Solution

The word *twice* tells us to multiply by 2.

	twice two hundred eleven
Translate.	$2(211)$
Multiply.	422

Note:
Exercise:

Problem: Translate and simplify: twice one hundred sixty-seven.

Solution:

2(167); 334

Multiply Whole Numbers in Applications

We will use the same strategy we used previously to solve applications of multiplication. First, we need to determine what we are looking for. Then we write a phrase that gives the information to find it. We then translate the phrase into math notation and simplify to get the answer. Finally, we write a sentence to answer the question.

Example:
Exercise:

Problem:

Humberto bought 4 sheets of stamps. Each sheet had 20 stamps. How many stamps did Humberto buy?

Solution:

We are asked to find the total number of stamps.

Write a phrase for the total.	the product of 4 and 20
Translate to math notation.	$4 \cdot 20$
Multiply.	$\begin{array}{r} 20 \\ \times 4 \\ \hline 80 \end{array}$
Write a sentence to answer the question.	Humberto bought 80 stamps.

Note:
Exercise:

Problem:

Valia donated water for the snack bar at her son's baseball game. She brought 6 cases of water bottles. Each case had 24 water bottles. How many water bottles did Valia donate?

Solution:

Valia donated 144 water bottles.

Note:
Exercise:

Problem:

Vanessa brought 8 packs of hot dogs to a family reunion. Each pack has 10 hot dogs. How many hot dogs did Vanessa bring?

Solution:

Vanessa bought 80 hot dogs.

Example:
Exercise:

Problem:

When Rena cooks rice, she uses twice as much water as rice. How much water does she need to cook 4 cups of rice?

Solution:
Solution

We are asked to find how much water Rena needs.

Write as a phrase.	twice as much as 4 cups
Translate to math notation.	$2 \cdot 4$
Multiply to simplify.	8
Write a sentence to answer the question.	Rena needs 8 cups of water for 4 cups of rice.

Note:
Exercise:

Problem:

Erin is planning her flower garden. She wants to plant twice as many dahlias as sunflowers. If she plants 14 sunflowers, how many dahlias does she need?

Solution:

Erin needs 28 dahlias.

Example:
Exercise:

Problem:

Van is planning to build a patio. He will have 8 rows of tiles, with 14 tiles in each row. How many tiles does he need for the patio?

Solution:
Solution

We are asked to find the total number of tiles.

Write a phrase.	the product of 8 and 14
Translate to math notation.	$8 \cdot 14$
Multiply to simplify.	$\begin{array}{r} {}^{3}14 \\ \times 8 \\ \hline 112 \end{array}$
Write a sentence to answer the question.	Van needs 112 tiles for his patio.

Note:
Exercise:

Problem:

Jane is tiling her living room floor. She will need 16 rows of tile, with 20 tiles in each row. How many tiles does she need for the living room floor?

Solution:

Jane needs 320 tiles.

Note:
Exercise:

Problem:

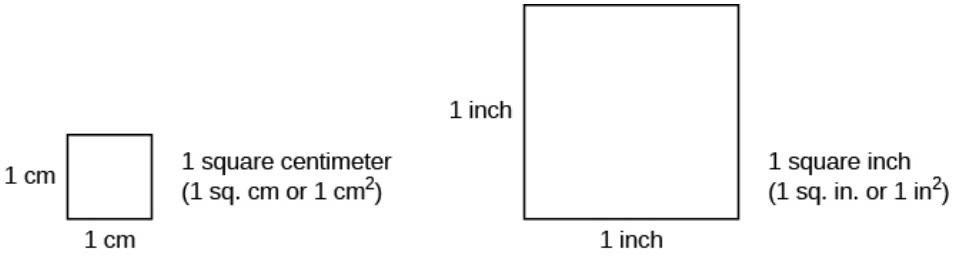
Yousef is putting shingles on his garage roof. He will need 24 rows of shingles, with 45 shingles in each row. How many shingles does he need for the garage roof?

Solution:

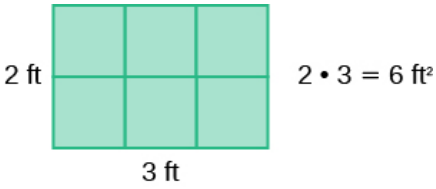
Yousef needs 1,080 tiles.

If we want to know the size of a wall that needs to be painted or a floor that needs to be carpeted, we will need to find its **area**. The area is a measure of the amount of surface that is covered by the shape. Area is measured in square units. We often use square inches, square feet, square centimeters, or square miles to

measure area. A square centimeter is a square that is one centimeter (cm.) on a side. A square inch is a square that is one inch on each side, and so on.



For a rectangular figure, the area is the product of the length and the width. [link] shows a rectangular rug with a length of 2 feet and a width of 3 feet. Each square is 1 foot wide by 1 foot long, or 1 square foot. The rug is made of 6 squares. The area of the rug is 6 square feet.



The area of a rectangle is the product of its length and its width, or 6 square feet.

Example:
Exercise:

Problem:

Jen's kitchen ceiling is a rectangle that measures 9 feet long by 12 feet wide. What is the area of Jen's kitchen ceiling?

Solution:

We are asked to find the area of the kitchen ceiling.

Write a phrase for the area.	the product of 9 and 12
Translate to math notation.	$9 \cdot 12$
Multiply.	$\begin{array}{r} {}^{1}12 \\ \times 9 \\ \hline 108 \end{array}$
Answer with a sentence.	The area of Jen's kitchen ceiling is 108 square feet.

Note:
Exercise:

Problem:

Zoila bought a rectangular rug. The rug is 8 feet long by 5 feet wide. What is the area of the rug?

Solution:

The area of the rug is 40 square feet.

We encourage you to go to Appendix B to take the Self Check for this section.

Note:
Access these online resources for additional instruction and practice with multiplying whole numbers.

- Multiplying Whole Numbers
- Multiplication with Partial Products
- Example of Multiplying by Whole Numbers

Key Concepts

- The Multiplication Property of Zero states that the product of any number and zero is zero. See [link].
- The Identity Property of Multiplication states that the product of any number and one is the number. See [link].
- The Commutative Property of Multiplication states that changing the order of the factors does not change their product. See [link].
- When numbers are multiplied vertically, it is important to line up digits in the same place value. See [link].
- When we multiply by a number with two or more digits, partial products can be calculated and then added. See [link].
- Zeros may be used as place holders in the partial products when multiplying, but they are not required as long as digits are recorded in the correct place value. Just as math notation can be translated into words, word phrases can be translated into math notation. Key words and phrases that indicate multiplication are *times*, *product*, and *twice*. See [link].
- Applications involving the multiplication of whole numbers requires identifying the problem, writing a phrase, translating to math notation, and then multiplying. See [link].
- Area. The area of a rectangle is the product of its length and its width. See [link].

Practice Makes Perfect

Use Multiplication Notation For the following exercises, translate from math notation to words.
Exercise:

Problem: 4×7

Solution:

four times seven; the product of four and seven

Exercise:

Problem: $(10)(25)$

Solution:

ten times twenty-five; the product of ten and twenty-five

Multiply Whole Numbers For the following exercises, fill in the missing values in each chart.

Exercise:

Problem:

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0		0	0	0	0		0
1	0	1	2	3			6	7	8	
2		2	4	6	8		12			18
3	0		6		12	15		21		27
4	0	4			16	20		28	32	
5	0	5	10	15			30		40	
6	0	6	12		24			42		54
7			14	21		35			56	63
8	0	8		24			48		64	
9	0	9	18		36	45			72	

Solution:

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Exercise:

Problem:

×	3	4	5	6	7	8	9
6							
7							
8							
9							

Solution:

×	3	4	5	6	7	8	9
6	18	24	30	36	42	48	54
7	21	28	35	42	49	56	63
8	24	32	40	48	56	64	72
9	27	36	45	54	63	72	81

Exercise:

Problem:

×	5	6	7	8	9
5					
6					
7					
8					
9					

Solution:

×	5	6	7	8	9
5	25	30	35	40	45
6	30	36	42	48	54
7	35	42	49	56	63
8	40	48	56	64	72
9	45	54	63	72	81

For the following exercises, multiply.

Exercise:

Problem: $0 \cdot 15$

Solution:

0

Exercise:

Problem: $(99)0$

Solution:

0

Exercise:

Problem: $1 \cdot 43$

Solution:

43

Exercise:

Problem: $(28)1$

Solution:

28

Exercise:

Problem: $1(240{,}055)$

Solution:

240,055

Exercise:

Problem: (a) $7 \cdot 6$ (b) $6 \cdot 7$

Solution:

(a) 42 (b) 42

Exercise:

Problem: $(79)(5)$

Solution:

395

Exercise:

Problem: $275 \cdot 6$

Solution:

1,650

Exercise:

Problem: $3{,}421 \times 7$

Solution:

23,947

Exercise:

Problem: $52(38)$

Solution:

1,976

Exercise:

Problem: $96 \cdot 73$

Solution:

7,008

Exercise:

Problem: 27×85

Solution:

2,295

Exercise:

Problem: $23 \cdot 10$

Solution:

230

Exercise:

Problem: $(100)(36)$

Solution:

360

Exercise:

Problem: $1{,}000(88)$

Solution:

88,000

Exercise:

Problem: $50 \times 1{,}000{,}000$

Solution:

50,000,000

Exercise:

Problem: 247×139

Solution:

34,333

Exercise:

Problem: $586(721)$

Solution:

422,506

Exercise:

Problem: $915 \cdot 879$

Solution:

804,285

Exercise:

Problem: $(104)(256)$

Solution:

26,624

Exercise:

Problem: $348(705)$

Solution:

245,340

Exercise:

Problem: $2{,}719 \times 543$

Solution:

1,476,417

Translate Word Phrases to Math Notation For the following exercises, translate and simplify.

Exercise:

Problem: the product of 18 and 33

Solution:

$18 \cdot 33$; 594

Exercise:

Problem: fifty-one times sixty-seven

Solution:

51(67); 3,417

Exercise:

Problem: twice 249

Solution:

2(249); 498

Exercise:

Problem: ten times three hundred seventy-five

Solution:

10(375); 3,750

Mixed Practice For the following exercises, simplify.

Exercise:

Problem: 38×37

Solution:

1,406

Exercise:

Problem: $415 - 267$

Solution:

148

Exercise:

Problem: $6{,}251 + 4{,}749$

Solution:

11,000

Exercise:

Problem: $(56)(204)$

Solution:

11,424

Exercise:

Problem: $947 \cdot 0$

Solution:

0

Exercise:

Problem: $15{,}382 + 1$

Solution:

15,383

For the following exercises, translate and simplify.

Exercise:

Problem: the difference of 50 and 18

Solution:

50 − 18; 32

Exercise:

Problem: twice 35

Solution:

2(35); 70

Exercise:

Problem: 20 more than 980

Solution:

20 + 980; 1,000

Exercise:

Problem: the product of 12 and 875

Solution:

12(875); 10,500

Exercise:

Problem: subtract 74 from 89

Solution:

89 − 74; 15

Exercise:

Problem: the sum of 3,075 and 950

Solution:

3,075 + 950; 4,025

Exercise:

Problem: 366 less than 814

Solution:

814 − 366; 448

Multiply Whole Numbers in Applications For the following exercises, solve.

Exercise:

Problem:

Party supplies Tim brought 9 six-packs of soda to a club party. How many cans of soda did Tim bring?

Solution:

Tim brought 54 cans of soda to the party.

Exercise:

Problem:

Field trip Seven school busses let off their students in front of a museum in Washington, DC. Each school bus had 44 students. How many students were there?

Solution:

There were 308 students.

Exercise:

Problem:

Charity Rey donated 15 twelve-packs of t-shirts to a homeless shelter. How many t-shirts did he donate?

Solution:

Rey donated 180 t-shirts.

Exercise:

Problem:

Recipe Stephanie is making punch for a party. The recipe calls for twice as much fruit juice as club soda. If she uses 10 cups of club soda, how much fruit juice should she use?

Solution:

Stephanie should use 20 cups of fruit juice.

Exercise:

Problem:

Government The United States Senate has twice as many senators as there are states in the United States. There are 50 states. How many senators are there in the United States Senate?

Solution:

There are 100 senators in the U.S. senate.

Exercise:

Problem:

Painting Jane is painting one wall of her living room. The wall is rectangular, 13 feet wide by 9 feet high. What is the area of the wall?

Solution:

The area of the wall is 117 square feet.

Exercise:

Problem:

Room size The meeting room in a senior center is rectangular, with length 42 feet and width 34 feet. What is the area of the meeting room?

Solution:

The area of the room is 1,428 square feet.

Exercise:

Problem:

NCAA basketball According to NCAA regulations, the dimensions of a rectangular basketball court must be 94 feet by 50 feet. What is the area of the basketball court?

Solution:

The area of the court is 4,700 square feet.

Everyday Math

Exercise:

Problem:

Stock market Javier owns 300 shares of stock in one company. On Tuesday, the stock price rose $12 per share. How much money did Javier's portfolio gain?

Solution:

Javier’s portfolio gained $3,600.

Writing Exercises

Exercise:

Problem:

How confident do you feel about your knowledge of the multiplication facts? If you are not fully confident, what will you do to improve your skills?

Solution:

Answers will vary.

Glossary

area
: a measure of the surface covered by a flat shape

Commutative Property of Multiplication
: states that changing the order of the factors does not change their product

factor
: a number being multiplied by one or more numbers

Identity Property of Multiplication
: rule that states that the product of any number and one is the number.

Multiplication Property of Zero
: rule that states that the product of any number and zero is zero

partial
: product each separate product of the digits in a multiplication problem

product
: the result obtained by multiplying two or more factors together

Dividing Whole Numbers
By the end of this section, you will be able to:

- Use division notation
- Model division of whole numbers
- Divide whole numbers
- Translate word phrases to math notation
- Divide whole numbers in applications

Division Notation

So far we have explored addition, subtraction, and multiplication. Now let's consider division. Suppose you have the 12 cookies in [link] and want to package them in bags with 4 cookies in each bag. How many bags would we need?

You might put 4 cookies in first bag, 4 in the second bag, and so on until you run out of cookies. Doing it this way, you would fill 3 bags.

In other words, starting with the 12 cookies, you would take away, or subtract, 4 cookies at a time. Division is a way to represent repeated subtraction just as multiplication represents repeated addition.

Instead of subtracting 4 repeatedly, we can write
Equation:

$$12 \div 4$$

We read this as *twelve divided by four* and the result is the **quotient** of 12 and 4. The quotient is 3 because we can subtract 4 from 12 exactly 3 times. We call the number being divided the

dividend and the number dividing it the **divisor**. In this case, the dividend is 12 and the divisor is 4.

In the past you may have used the notation $4\overline{)12}$, but this division also can be written as $12 \div 4,\ 12/4,\ \frac{12}{4}$. In each case the 12 is the dividend and the 4 is the divisor.

Note:
Operation Symbols for Division
To represent and describe division, we can use symbols and words.

Operation	Notation	Expression	Read as	Result
Division	$\div$	$12 \div 4$	Twelve divided by four	the quotient of 12 and 4
	$\frac{a}{b}$	$\frac{12}{4}$		
	$\overline{)}$	$4\overline{)12}$		

Divide Whole Numbers

We said that addition and subtraction are inverse operations because one undoes the other. Similarly, division is the inverse operation of multiplication. We know $12 \div 4 = 3$ because $3 \cdot 4 = 12$. Knowing all the multiplication number facts is very important when doing division.

We check our answer to division by multiplying the quotient by the divisor to determine if it equals the dividend. In [link], we know $24 \div 8 = 3$ is correct because $3 \cdot 8 = 24$.

Example:
Exercise:

Problem: Divide. Then check by multiplying. (a) $42 \div 6$ (b) $\frac{72}{9}$ (c) $7\overline{)63}$

Solution:

(a)

	$42 \div 6$
Divide 42 by 6.	7
Check by multiplying.	$7 \cdot 6$
	$42\checkmark$

(b)

	$\frac{72}{9}$
Divide 72 by 9.	8
Check by multiplying.	$8 \cdot 9$
	$72 \checkmark$

(c)

	$7\overline{)63}$
Divide 63 by 7.	9
Check by multiplying.	$9 \cdot 7$
	$63 \checkmark$

Note:
Exercise:

Problem: Divide. Then check by multiplying:

(a) $54 \div 6$ (b) $\frac{27}{9}$

Solution:

(a) 9 (b) 3

Note:
Exercise:

Problem: Divide. Then check by multiplying:

(a) $\frac{36}{9}$ (b) $8\overline{)40}$

Solution:

(a) 4 (b) 5

What is the quotient when you divide a number by itself?
Equation:

$$\frac{15}{15} = 1 \text{ because } 1 \cdot 15 = 15$$

Dividing any number (except 0) by itself produces a quotient of 1. Also, any number divided by 1 produces a quotient of the number. These two ideas are stated in the **Division Properties of One**.

Note:
Division Properties of One

Any number (except 0) divided by itself is one.	$a \div a = 1$
Any number divided by one is the same number.	$a \div 1 = a$

Example:
Exercise:

Problem: Divide. Then check by multiplying:

(a) $11 \div 11$

(b) $\frac{19}{1}$

(c) $1\overline{)7}$

Solution:
Solution

(a)

	$11 \div 11$
A number divided by itself is 1.	1
Check by multiplying.	$1 \cdot 11$
	11✓

(b)

	$\frac{19}{1}$
A number divided by 1 equals itself.	19
Check by multiplying.	$19 \cdot 1$
	19✓

(c)

	$1\overline{)7}$
A number divided by 1 equals itself.	7
Check by multiplying.	$7 \cdot 1$
	7✓

Note:
Exercise:

Problem: Divide. Then check by multiplying:

(a) $14 \div 14$ (b) $\frac{27}{1}$

Solution:

(a) 1 (b) 27

Note:
Exercise:

Problem: Divide. Then check by multiplying:

(a) $\frac{16}{1}$ (b) $1\overline{)4}$

Solution:

(a) 16 (b) 4

Suppose we have $\$0$, and want to divide it among 3 people. How much would each person get? Each person would get $\$0$. Zero divided by any number is 0.

Now suppose that we want to divide $\$10$ by 0. That means we would want to find a number that we multiply by 0 to get 10. This cannot happen because 0 times any number is 0. Division by zero is said to be *undefined*.

These two ideas make up the **Division Properties of Zero**.

Note:
Division Properties of Zero

Zero divided by any number is 0.	$0 \div a = 0$
Dividing a number by zero is undefined.	$a \div 0$ undefined

Another way to explain why division by zero is undefined is to remember that division is really repeated subtraction. How many times can we take away 0 from 10? Because subtracting 0 will never change the total, we will never get an answer. So we cannot divide a number by 0.

Example:
Exercise:

Problem: Divide. Check by multiplying: $(\text{a})\ 0 \div 3\ (\text{b})\ 10/0$.

Solution:

(a)

	$0 \div 3$
Zero divided by any number is zero.	0
Check by multiplying.	$0 \cdot 3$
	0✓

(b)

	$10/0$
Division by zero is undefined.	undefined

When the divisor or the dividend has more than one digit, it is usually easier to use the $4\overline{)12}$ notation. This process is called long division. Let’s work through the process by dividing 78 by 3.

Divide the first digit of the dividend, 7, by the divisor, 3.

The divisor 3 can go into 7 two times since $2 \times 3 = 6$.
Write the 2 above the 7 in the quotient.

$$3\overline{)78}\quad \text{quotient: } 2$$

Multiply the 2 in the quotient by 3 and
write the product, 6, under the 7.

$$\begin{array}{r} 2 \\ 3\overline{)78} \\ 6 \end{array}$$

Subtract that product from the first digit in the dividend.

Subtract $7 - 6$. Write the difference, 1,
under the first digit in the dividend.

$$\begin{array}{r} 2 \\ 3\overline{)78} \\ 6 \\ \hline 1 \end{array}$$

Bring down the next digit of the dividend. Bring down the 8.

$$\begin{array}{r} 2 \\ 3\overline{)78} \\ 6 \\ \hline 18 \end{array}$$

Divide 18 by the divisor, 3. The divisor 3 goes into 18 six times.

$$\begin{array}{r} 26 \\ 3\overline{)78} \\ 6 \\ \hline 18 \end{array}$$

Write 6 in the quotient above the 8.

Multiply the 6 in the quotient by the divisor and
write the product, 18, under the dividend. Subtract 18 from 18.

$$\begin{array}{r} 26 \\ 3\overline{)78} \\ 6 \\ \hline 18 \\ 18 \\ \hline 0 \end{array}$$

We would repeat the process until there are no more digits in the dividend to bring down. In this problem, there are no more digits to bring down, so the division is finished.

Equation:

$$\text{So } 78 \div 3 = 26.$$

Check by multiplying the quotient times the divisor to get the dividend. Multiply 26×3 to make sure that product equals the dividend, 78.

Equation:

$$\begin{array}{r} {\scriptstyle 1} \\ 26 \\ \times 3 \\ \hline 78\checkmark \end{array}$$

It does, so our answer is correct.

Note:
Divide whole numbers.
Step 1. Divide the first digit of the dividend by the divisor.

- If the divisor is larger than the first digit of the dividend, divide the first two digits of the dividend by the divisor, and so on.

Step 2. Write the quotient above the dividend.
Step 3. Multiply the quotient by the divisor and write the product under the dividend.
Step 4. Subtract that product from the dividend.
Step 5. Bring down the next digit of the dividend.
Step 6. Repeat from Step 1 until there are no more digits in the dividend to bring down.
Step 7. Check by multiplying the quotient times the divisor.

Example:
Exercise:

Problem: Divide $2{,}596 \div 4$. Check by multiplying:

Solution:

$2{,}596 \div 4 = 649.$

Check by multiplying.

$$\begin{array}{r} {\scriptstyle 1\ 3} \\ 649 \\ \times\ 4 \\ \hline 2{,}596\checkmark \end{array}$$

It equals the dividend, so our answer is correct.

Note:
Exercise:

Problem: Divide. Then check by multiplying:

$2{,}636 \div 4$

Solution:

659

Note:
Exercise:

Problem: Divide. Then check by multiplying:

$2{,}716 \div 4$

Solution:

679

Example:
Exercise:

Problem: Divide $4{,}506 \div 6$. Check by multiplying:

Solution:

$6\overline{)4{,}506}$

First we try to divide 6 into 4.

$6\overline{)4506}$

Since that won't work, we try 6 into 45.

There are 7 sixes in 45. We write the 7 over the 5.

$$\begin{array}{r} 7 \\ 6\overline{)4506} \end{array}$$

Multiply the 7 by 6 and subtract this product from 45.

$$\begin{array}{r} 7 \\ 6\overline{)4506} \\ \underline{42} \\ 3 \end{array}$$

Now bring down the 0 and repeat these steps.

$$\begin{array}{r} 75 \\ 6\overline{)4506} \\ \underline{42} \\ 30 \end{array}$$

There are 5 sixes in 30. Write the 5 over the 0.

Multiply the 5 by 6 and subtract this product from 30.

$$\begin{array}{r} 75 \\ 6\overline{)4506} \\ \underline{42} \\ 30 \\ \underline{30} \\ 0 \end{array}$$

Now bring down the 6 and repeat these steps.

$$\begin{array}{r} 751 \\ 6\overline{)4506} \\ \underline{42} \\ 30 \\ \underline{30} \\ 06 \end{array}$$

There is 1 six in 6. Write the 1 over the 6.

Multiply the 1 by 6 and subtract this product from 6.

$$\begin{array}{r} 751 \\ 6\overline{)4506} \\ \underline{42} \\ 30 \\ \underline{30} \\ 06 \\ \underline{6} \\ 0 \end{array}$$

Check by multiplying.

$$\begin{array}{r} {}^{3} \\ 751 \\ \times\ 6 \\ \hline 4{,}506\checkmark \end{array}$$

It equals the dividend, so our answer is correct.

Note:
Exercise:

Problem: Divide. Then check by multiplying: $4{,}305 \div 5$.

Solution:

861

Note:
Exercise:

Problem: Divide. Then check by multiplying: $3{,}906 \div 6$.

Solution:

651

Note:
Exercise:

Problem: Divide. Then check by multiplying: $4{,}928 \div 7$.

Solution:

704

Note:
Exercise:

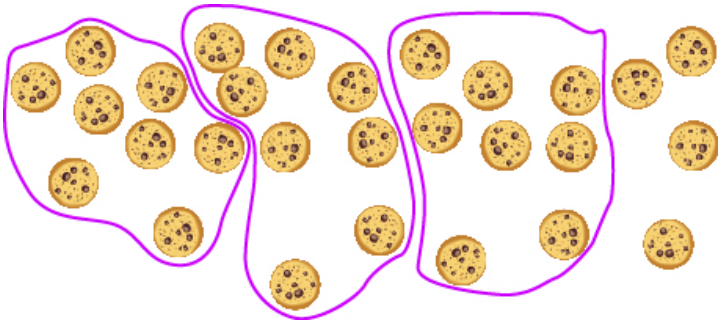
Problem: Divide. Then check by multiplying: $5{,}663 \div 7$.

Solution:

809

So far all the division problems have worked out evenly. For example, if we had 24 cookies and wanted to make bags of 8 cookies, we would have 3 bags. But what if there were 28 cookies and we wanted to make bags of 8? Start with the 28 cookies as shown in [link].

Try to put the cookies in groups of eight as in [link].



There are 3 groups of eight cookies, and 4 cookies left over. We call the 4 cookies that are left over the remainder and show it by writing R4 next to the 3. (The R stands for remainder.)

To check this division we multiply 3 times 8 to get 24, and then add the remainder of 4.
Equation:

$$\begin{array}{r} 3 \\ \times 8 \\ \hline 24 \\ +4 \\ \hline 28 \end{array}$$

Example:
Exercise:

Problem: Divide $1{,}439 \div 4$. Check by multiplying.

Solution:
Solution

$$4\overline{)1439}$$

First we try to divide 4 into 1.

Since that won't work, we try 4 into 14.

There are 3 fours in 14. We write the 3 over the 4.

$$\begin{array}{r} 3 \\ 4\overline{)1439} \end{array}$$

Multiply the 3 by 4 and subtract this product from 14.

$$\begin{array}{r} 3 \\ 4\overline{)1439} \\ \underline{12} \\ 2 \end{array}$$

Now bring down the 3 and repeat these steps.

There are 5 fours in 23. Write the 5 over the 3.

Multiply the 5 by 4 and subtract this product from 23.

$$\begin{array}{r} 35 \\ 4\overline{)1439} \\ \underline{12}\downarrow \\ 23 \\ \underline{20} \\ 3 \end{array}$$

Now bring down the 9 and repeat these steps.

There are 9 fours in 39. Write the 9 over the 9.

Multiply the 9 by 4 and subtract this product from 39.

There are no more numbers to bring down, so we are done.

The remainder is 3.

$$\begin{array}{r} 359\text{R}3 \\ 4\overline{)1439} \\ \underline{12} \\ 23 \\ \underline{20}\downarrow \\ 39 \\ \underline{36} \\ 3 \end{array}$$

Check by multiplying.

$$\begin{array}{rl} \overset{2\ 2}{359} & \text{quotient} \\ \times\ 4 & \text{divisor} \\ \hline 1{,}436 & \\ +\ 3 & \text{remainder} \\ \hline 1{,}439\checkmark & \end{array}$$

So $1{,}439 \div 4$ is 359 with a remainder of 3. Our answer is correct.

Note:
Exercise:

Problem: Divide. Then check by multiplying: $3{,}812 \div 8$.

Solution:

476 with a remainder of 4

Note:
Exercise:

Problem: Divide. Then check by multiplying: $4{,}319 \div 8$.

Solution:

539 with a remainder of 7

Example:
Exercise:

Problem: Divide and then check by multiplying: $1{,}461 \div 13$.

Solution:
Solution

$13\overline{)1461}$

First we try to divide 13 into 1.
Since that won't work, we try 13 into 14.
There is 1 thirteen in 14. We write the 1 over the 4.

$$\begin{array}{r} 1 \\ 13\overline{)1461} \end{array}$$

Multiply the 1 by 13 and subtract this product from 14.

$$\begin{array}{r} 1 \\ 13\overline{)1461} \\ \underline{13} \\ 1 \end{array}$$

Now bring down the 6 and repeat these steps.
There is 1 thirteen in 16. Write the 1 over the 6.
Multiply the 1 by 13 and subtract this product from 16.

$$\begin{array}{r} 11 \\ 13\overline{)1461} \\ \underline{13}\downarrow \\ 16 \\ \underline{13} \\ 3 \end{array}$$

Now bring down the 1 and repeat these steps.
There are 2 thirteens in 31. Write the 2 over the 1.
Multiply the 2 by 13 and subtract this product from 31.

$$\begin{array}{r} 112\text{R}5 \\ 13\overline{)1461} \\ \underline{13} \\ 16 \\ \underline{13}\downarrow \\ 31 \\ \underline{26} \\ 5 \end{array}$$

There are no more numbers to bring down, so we are done. The remainder is 5.
1,461 ÷ 13 is 112 with a remainder of 5.

Check by multiplying.

$$\begin{array}{rl} 112 & \text{quotient} \\ \times\ 13 & \text{divisor} \\ \hline 336 & \\ 1{,}120 & \\ +\ \ 5 & \text{remainder} \\ \hline 1{,}461 & \checkmark \end{array}$$

Our answer is correct.

Note:
Exercise:

Problem: Divide. Then check by multiplying: 1,493 ÷ 13.

Solution:

114 R11

Note:
Exercise:

Problem: Divide. Then check by multiplying: $1{,}461 \div 12$.

Solution:

121 R9

Example:
Exercise:

Problem: Divide and check by multiplying: $74{,}521 \div 241$.

Solution:
Solution

$241\overline{)74521}$

First we try to divide 241 into 7.
Since that won't work, we try 241 into 74.
That still won't work, so we try 241 into 745.
Since 2 divides into 7 three times, we try 3.
Since $3 \times 241 = 723$, we write the 3 over the 5 in 745.
Note that 4 would be too large because $4 \times 241 = 964$,
which is greater than 745.

Multiply the 3 by 241 and subtract this product from 745.

$$\begin{array}{r} 3 \\ 241\overline{)74521} \\ \underline{723} \\ 22 \end{array}$$

Now bring down the 2 and repeat these steps.
241 does not divide into 222.
We write a 0 over the 2 as a placeholder and then continue.

$$\begin{array}{r} 30 \\ 241\overline{)74521} \\ \underline{723} \\ 222 \end{array}$$

Now bring down the 1 and repeat these steps.
Try 9. Since $9 \times 241 = 2{,}169$, we write the 9 over the 1.
Multiply the 9 by 241 and subtract this product from 2,221.

$$\begin{array}{r} 309 \text{ R52} \\ 241\overline{)74521} \\ \underline{723} \\ 2221 \\ \underline{2169} \\ 52 \end{array}$$

There are no more numbers to bring down, so we are finished.
The remainder is 52.

So 74,521 ÷ 241 is 309 with a remainder of 52.

Check by multiplying.

$$\begin{array}{rl} \overset{3}{} & \\ 309 & \text{quotient} \\ \times\ 241 & \text{divisor} \\ \hline 309 & \\ 12{,}360 & \\ \underline{61{,}800} & \\ 72{,}469 & \\ +\ 52 & \text{remainder} \\ \hline 74{,}3521 & \checkmark \end{array}$$

Sometimes it might not be obvious how many times the divisor goes into digits of the dividend. We will have to guess and check numbers to find the greatest number that goes into the digits without exceeding them.

Note:
Exercise:

Problem: Divide. Then check by multiplying: $78{,}641 \div 256$.

Solution:

307 R49

Note:
Exercise:

Problem: Divide. Then check by multiplying: $76{,}461 \div 248$.

Solution:

308 R77

Translate Word Phrases to Math Notation

Earlier in this section, we translated math notation for division into words. Now we'll translate word phrases into math notation. Some of the words that indicate division are given in [link].

Operation	Word Phrase	Example	Expression
Division	divided by quotient of divided into	12 divided by 4 the quotient of 12 and 4 4 divided into 12	$12 \div 4$ $\frac{12}{4}$ $12/4$ $4\overline{)12}$

Example:
Exercise:

Problem: Translate and simplify: the quotient of 51 and 17.

Solution:
Solution

The word *quotient* tells us to divide.

	the quotient of 51 and 17
Translate.	$51 \div 17$
Divide	3

We could just as correctly have translated *the quotient of* 51 *and* 17 using the notation $17\overline{)51}$ or $\frac{51}{17}$.

Note:
Exercise:

Problem: Translate and simplify: the quotient of 91 and 13.

Solution:

91 ÷ 13; 7

Note:
Exercise:

Problem: Translate and simplify: the quotient of 52 and 13.

Solution:

52 ÷ 13; 4

Divide Whole Numbers in Applications

We will use the same strategy we used in previous sections to solve applications. First, we determine what we are looking for. Then we write a phrase that gives the information to find it. We then translate the phrase into math notation and simplify it to get the answer. Finally, we write a sentence to answer the question.

Example:
Exercise:

Problem:

Cecelia bought a 160-ounce box of oatmeal at the big box store. She wants to divide the 160 ounces of oatmeal into 8-ounce servings. She will put each serving into a plastic bag so she can take one bag to work each day. How many servings will she get from the big box?

Solution:

We are asked to find the how many servings she will get from the big box.

Write a phrase	160 ounces divided by 8 ounces
Translate to math notation	$160 \div 8$
Simplify by dividing.	20
Write a sentence to answer the question.	Cecelia will get 20 servings from the big box.

Note:
Exercise:

Problem:

Marcus is setting out animal crackers for snacks at the preschool. He wants to put 9 crackers in each cup. One box of animal crackers contains 135 crackers. How many cups can he fill from one box of crackers?

Solution:

Marcus can fill 15 cups.

Note:
Exercise:

Problem:

Andrea is making bows for the girls in her dance class to wear at the recital. Each bow takes 4 feet of ribbon, and 36 feet of ribbon are on one spool. How many bows can Andrea make from one spool of ribbon?

Solution:

Andrea can make 9 bows.

We encourage you to go to Appendix B to complete the Self Check for this section.

Note:
Access these online resources for additional instruction and practice with dividing whole numbers.

- Dividing Whole Numbers
- Dividing Whole Numbers No Remainder
- Dividing Whole Numbers With Remainder

Key Concepts

- Division and multiplication are inverse operations so the result of a division problem can be checked by multiplying the divisor by the dividend. See [link].
- The Division Properties of One state that any number $(\text{except } 0)$ divided by itself is one and any number divided by one is the same number. See [link].
- The Division Properties of Zero state that zero divided by any number is 0 and dividing a number by zero is undefined. See [link].
- Long division can be used to divide large numbers. See [link].
- Dividing by a divisor that does not go evenly into the dividend results in a remainder. See [link].
- Just as math notation can be translated into words, word phrases can be translated into math notation. Key words and phrases that indicate division are *divided by, quotient of, total*, and *divided into*. See [link].
- Applications involving the division of whole numbers requires identifying the problem, writing a phrase, translating to math notation, and then dividing. See [link].

Practice Makes Perfect

Divide Whole Numbers For the following exercises, divide. Then check by multiplying.
Exercise:

Problem: $18 \div 2$

Solution:

9

Exercise:

Problem: $\frac{27}{3}$

Solution:

9

Exercise:

Problem: $4\overline{)28}$

Solution:

7

Exercise:

Problem: $\frac{45}{5}$

Solution:

9

Exercise:

Problem: $72/8$

Solution:

9

Exercise:

Problem: $\frac{35}{7}$

Solution:

5

Exercise:

Problem: $15\overline{)15}$

Solution:

1

Exercise:

Problem: $43 \div 43$

Solution:

1

Exercise:

Problem: $\frac{23}{1}$

Solution:

23

Exercise:

Problem: $19 \div 1$

Solution:

19

Exercise:

Problem: $0 \div 4$

Solution:

0

Exercise:

Problem: $\frac{5}{0}$

Solution:

undefined

Exercise:

Problem: $\frac{26}{0}$

Solution:

undefined

Exercise:

Problem: $12\overline{)0}$

Solution:

0

Exercise:

Problem: $72 \div 3$

Solution:

24

Exercise:

Problem: $\frac{96}{8}$

Solution:

12

Exercise:

Problem: $5\overline{)465}$

Solution:

93

Exercise:

Problem: $924 \div 7$

Solution:

132

Exercise:

Problem: $\frac{5{,}226}{6}$

Solution:

871

Exercise:

Problem: $4\overline{)31{,}324}$

Solution:

7,831

Exercise:

Problem: $7{,}209 \div 3$

Solution:

2,403

Exercise:

Problem: $5{,}406 \div 6$

Solution:

901

Exercise:

Problem: $4\overline{)2{,}816}$

Solution:

704

Exercise:

Problem: $\frac{91{,}881}{9}$

Solution:

10,209

Exercise:

Problem: $2{,}470 \div 7$

Solution:

352 R6

Exercise:

Problem: $8\overline{)55{,}305}$

Solution:

6,913 R1

Exercise:

Problem: $\frac{431{,}174}{5}$

Solution:

86,234 R4

Exercise:

Problem: $130{,}016 \div 3$

Solution:

43,338 R2

Exercise:

Problem: $15\overline{)5{,}735}$

Solution:

382 R5

Exercise:

Problem: $56{,}883 \div 67$

Solution:

849

Exercise:

Problem: $\frac{30{,}144}{314}$

Solution:

96

Exercise:

Problem: $273\overline{)542{,}195}$

Solution:

1,986 R17

Mixed Practice For the following exercises, simplify.
Exercise:

Problem: $15\,(204)$

Solution:

3,060

Exercise:

Problem: $256 - 184$

Solution:

72

Exercise:

Problem: $719 + 341$

Solution:

1,060

Exercise:

Problem: $25\overline{)875}$

Solution:

35

Translate Word Phrases to Algebraic Expressions For the following exercises, translate and simplify.
Exercise:

Problem: the quotient of 45 and 15

Solution:

$45 \div 15$; 3

Exercise:

Problem: the quotient of 288 and 24

Solution:

288 ÷ 24; 12

Divide Whole Numbers in Applications For the following exercises, solve.

Exercise:

Problem:

Trail mix Ric bought 64 ounces of trail mix. He wants to divide it into small bags, with 2 ounces of trail mix in each bag. How many bags can Ric fill?

Solution:

Ric can fill 32 bags.

Exercise:

Problem:

There are 125 students in an astronomy class. The professor assigns them into groups of 5. How many groups of students are there?

Solution:

There are 25 groups.

Exercise:

Problem:

One roll of plastic wrap is 48 feet long. Marta uses 3 feet of plastic wrap to wrap each cake she bakes. How many cakes can she wrap from one roll?

Solution:

Marta can wrap 16 cakes from 1 roll.

Mixed Practice For the following exercises, solve.

Exercise:

Problem:

Susana’s hybrid car gets 45 miles per gallon. Her son’s truck gets 17 miles per gallon. What is the difference in miles per gallon between Jane’s car and her son’s truck?

Solution:

The difference is 28 miles per gallon.

Exercise:

Problem:

The 45 students in a Geology class will go on a field trip, using the college's vans. Each van can hold 9 students. How many vans will they need for the field trip?

Solution:

They will need 5 vans for the field trip

Exercise:

Problem:

Bill hiked 8 miles on the first day of his backpacking trip, 14 miles the second day, 11 miles the third day, and 17 miles the fourth day. What is the total number of miles Bill hiked?

Solution:

Bill hiked 50 miles

Exercise:

Problem:

LaVonne treats 12 patients each day in her dental office. Last week she worked 4 days. How many patients did she treat last week?

Solution:

LaVonne treated 48 patients last week.

Everyday Math

Exercise:

Problem:

Contact lenses Jenna puts in a new pair of contact lenses every 14 days. How many pairs of contact lenses does she need for 365 days?

Solution:

Jenna uses 26 pairs of contact lenses, but there is 1 day left over, so she needs 27 pairs for 365 days.

Chapter 1 Review Exercises

Introduction to Whole Numbers

Identify Counting Numbers and Whole Numbers For the following exercises, determine which of the following are (a) counting numbers (b) whole numbers.

Exercise:

Problem: $0, 2, 99$

Solution:

(a) 2, 99 (b) 0, 2, 99

Identify the Place Value of a Digit For the following exercises, find the place value of the given digits.

Exercise:

Problem: $472{,}981$

(a) 8

(b) 4

(c) 1

(d) 7

(e) 2

Solution:

(a) tens (b) hundred thousands (c) ones (d) thousands (e) ten thousands

Use Place Value to Name Whole Numbers For the following exercises, name each number in words.

Exercise:

Problem: $5{,}280$

Solution:

Five thousand two hundred eighty

Exercise:

Problem: $5{,}012{,}582$

Solution:

Five million twelve thousand five hundred eighty-two

Use Place Value to Write Whole Numbers For the following exercises, write as a whole number using digits.

Exercise:

Problem: fifteen thousand, two hundred fifty-three

Solution:

15,253

Exercise:

Problem: three hundred forty million, nine hundred twelve thousand, sixty-one

Solution:

340,912,061

Round Whole Numbers For the following exercises, round to the nearest ten.

Exercise:

Problem: 412

Solution:

410

Exercise:

Problem: 3,556

Solution:

3,560

For the following exercises, round to the nearest hundred.

Exercise:

Problem: 38,975

Solution:

39,000

Exercise:

Problem: $81{,}486$

Solution:

81,500

Add Whole Numbers

For the following exercises, add.

Exercise:

Problem: $(\text{a})\ 0+19\ (\text{b})\ 19+0$

Solution:

(a) 19 (b) 19

Exercise:

Problem: $(\text{a})\ 7+6\ (\text{b})\ 6+7$

Solution:

(a) 13 (b) 13

Exercise:

Problem: $44+35$

Solution:

82

Exercise:

Problem: $96+58$

Solution:

154

Exercise:

Problem: $7{,}281+12{,}546$

Solution:

19,827

Translate Word Phrases to Math Notation For the following exercises, translate each phrase into math notation and then simplify.

Exercise:

Problem: the sum of 30 and 12

Solution:

30 + 12; 42

Exercise:

Problem: 25 more than 39

Solution:

39 + 25; 64

Add Whole Numbers in Applications For the following exercises, solve.

Exercise:

Problem:

Shopping for an interview Nathan bought a new shirt, tie, and slacks to wear to a job interview. The shirt cost $24, the tie cost $14, and the slacks cost $38. What was Nathan's total cost?

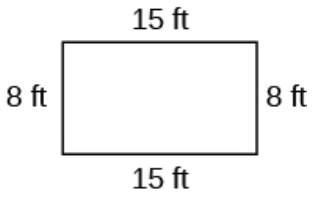
Solution:

$76

For the following exercises, find the perimeter of each figure.

Exercise:

Problem:



Solution:

46 feet

Subtract Whole Numbers

Subtract Whole Numbers For the following exercises, subtract and then check by adding.

Exercise:

Problem: $8 - 5$

Solution:

3

Exercise:

Problem: $23 - 9$

Solution:

14

Exercise:

Problem: $82 - 59$

Solution:

23

Exercise:

Problem: $539 - 217$

Solution:

322

Exercise:

Problem: $1{,}020 - 640$

Solution:

380

Exercise:

Problem: $10{,}000 - 15$

Solution:

9,985

Translate Word Phrases to Math Notation For the following exercises, translate and simplify.
Exercise:

Problem: the difference of nineteen and thirteen

Solution:

19 − 13; 6

Exercise:

Problem: seventy-four decreased by eight

Solution:

74 − 8; 66

Subtract Whole Numbers in Applications For the following exercises, solve.
Exercise:

Problem:

Temperature The high temperature in Peoria one day was 86 degrees Fahrenheit and the low temperature was 28 degrees Fahrenheit. What was the difference between the high and low temperatures?

Solution:

58 degrees Fahrenheit

Multiply Whole Numbers

Multiply Whole Numbers For the following exercises, fill in the missing values in each chart.
Exercise:

Problem:

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0		0		0	0
1	0	1	2		4	5	6	7		9
2	0		4		8	10		14	16	
3		3		9			18		24	
4	0	4		12			24			36
5	0	5	10		20		30	35	40	45
6			12	18			36	42		54
7	0	7		21		35			56	63
8	0	8	16		32		48		64	
9			18	27	36			63	72	

Solution:

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

For the following exercises, multiply.

Exercise:

Problem: $0 \cdot 14$

Solution:

0

Exercise:

Problem: $1 \cdot 99$

Solution:

99

Exercise:

Problem: a) $7 \cdot 4$; b) $4 \cdot 7$

Solution:

(a) 28 (b) 28

Exercise:

Problem: $9{,}261 \times 3$

Solution:

27,783

Exercise:

Problem: $64 \cdot 10$

Solution:

640

Exercise:

Problem: 162×493

Solution:

79,866

Exercise:

Problem: $3{,}624 \times 517$

Solution:

1,873,608

Translate Word Phrases to Math Notation For the following exercises, translate and simplify.
Exercise:

Problem: the product of 15 and 28

Solution:

15(28); 420

Exercise:

Problem: twice 575

Solution:

2(575); 1,150

Multiply Whole Numbers in Applications For the following exercises, solve.
Exercise:

Problem:

Gardening Geniece bought 8 packs of marigolds to plant in her yard. Each pack has 6 flowers. How many marigolds did Geniece buy?

Solution:

48 marigolds

Exercise:

Problem:

Multiplex There are twelve theaters at the multiplex and each theater has 150 seats. What is the total number of seats at the multiplex?

Solution:

1,800 seats

Divide Whole Numbers

Divide Whole Numbers For the following exercises, divide. Then check by multiplying.

Exercise:

Problem: $14 \div 2$

Solution:

7

Exercise:

Problem: $52 \div 4$

Solution:

13

Exercise:

Problem: $\frac{97}{1}$

Solution:

97

Exercise:

Problem: $100 \div 0$

Solution:

undefined

Exercise:

Problem: $3828 \div 6$

Solution:

638

Exercise:

Problem: $\frac{7505}{25}$

Solution:

300 R5

Translate Word Phrases to Math Notation For the following exercises, translate and simplify.
Exercise:

Problem: the quotient of 64 and 16

Solution:

64 ÷ 16; 4

Divide Whole Numbers in Applications For the following exercises, solve.
Exercise:

Problem:

Ribbon One spool of ribbon is 27 feet. Lizbeth uses 3 feet of ribbon for each gift basket that she wraps. How many gift baskets can Lizbeth wrap from one spool of ribbon?

Solution:

9 baskets

Chapter 1 Practice Test

Exercise:

Problem: Determine which of the following numbers are

(a) counting numbers

(b) whole numbers.

$0, 4, 87$

Solution:

(a) 4, 87 (b) 0, 4, 8

Exercise:

Problem: Find the place value of the given digits in the number $549{,}362$.

(a) 9

(b) 6

(c) 2

(d) 5

Solution
(a) 9 thousands (b) 6 tens (c) 2 ones (d) 5 hundred thousands
Exercise:

Problem: Write each number as a whole number using digits.

(a) six hundred thirteen

(b) fifty-five thousand two hundred eight

Solution:

(a) 613 (b) 55,208

Exercise:

Problem: Round $25{,}849$ to the nearest hundred.

Solution
25,800

Exercise:

Problem: $45 + 23$

Solution:

68

Exercise:

Problem: $65 - 42$

Solution
23
Exercise:

Problem: $85 \div 5$

Solution:

17

Exercise:

Problem: $1{,}000 \times 8$

Solution
8,000
Exercise:

Problem: $90 - 58$

Solution:

32

Exercise:

Problem: $73 + 89$

Solution
162
Exercise:

Problem: $(0)(12{,}675)$

Solution:

0

Exercise:

Problem: $634 + 255$

Solution
889
Exercise:

Problem: $\frac{0}{9}$

Solution:

0

Exercise:

Problem: $8\overline{)128}$

Solution

16

Exercise:

Problem: $145 - 79$

Solution:

66

Exercise:

Problem: $7 \cdot 475$

Solution:

3,325

Exercise:

Problem: $35(14)$

Solution:

490

Exercise:

Problem: $\frac{26}{0}$

Solution

Undefined

Exercise:

Problem: $733 - 291$

Solution:

442

Exercise:

Problem: $495 \div 45$

Solution:

11

Exercise:

Problem: 52×983

Solution
51,116

Translate each phrase to math notation and then simplify.
Exercise:

Problem: The sum of 16 and 58

Solution:

16 + 58; 74

Exercise:

Problem: The product of 9 and 15

Solution
9 x 15 = 135
Exercise:

Problem: The difference of 32 and 18

Solution:

32 − 18; 14

Exercise:

Problem: The quotient of 63 and 21

Solution
63/21 = 3
Exercise:

Problem: Twice 524

Solution:

2(524); 1,048

Exercise:

Problem: 29 more than 32

Solution
29 + 32 = 61
Exercise:

Problem: 50 less than 300

Solution:

300 − 50; 250

For the following exercises, solve.
Exercise:

Problem:

Last month, Stan's take-home pay was $3,816 and his expenses were $3,472. How much of his take-home pay did Stan have left after he paid his expenses?

Solution:

Stan had $344 left.

Exercise:

Problem:

Clayton walked 12 blocks to his mother's house, 6 blocks to the gym, and 9 blocks to the grocery store before walking the last 3 blocks home. What was the total number of blocks that Clayton walked?

Solution:

Clayton walked 30 blocks.

Glossary

dividend
: number being divided by another

Division Properties of One
: rules that state that dividing any number (except 0) by itself produces a quotient of 1 or dividing any number by 1 is the number

Division Properties of Zero
: rules that state that zero divided by any number is 0 and any number divided by zero is undefined

divisor
: a number doing the dividing in a division problem

quotient
: result obtained when dividing one number by another